CSE 250
Data Structures

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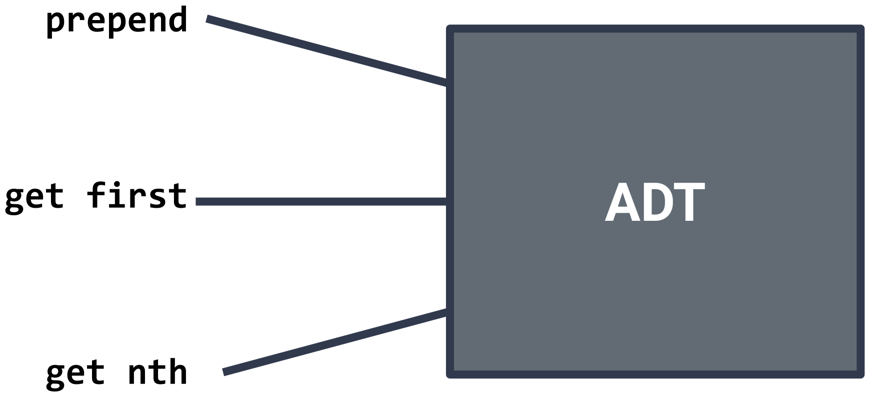
Lec 04: Intro to Complexity

Announcements and Feedback

- Join Piazza! (Link on course website)
- Normal recitations (w/attendance) begin next week
- Academic Integrity Quiz due 2/4 @ 11:59PM **(MUST GET 100%)**
- PA0 due 2/4 @ 11:59PM **(MUST GET 100%)**
- WA1 due 2/4 @ 11:59PM

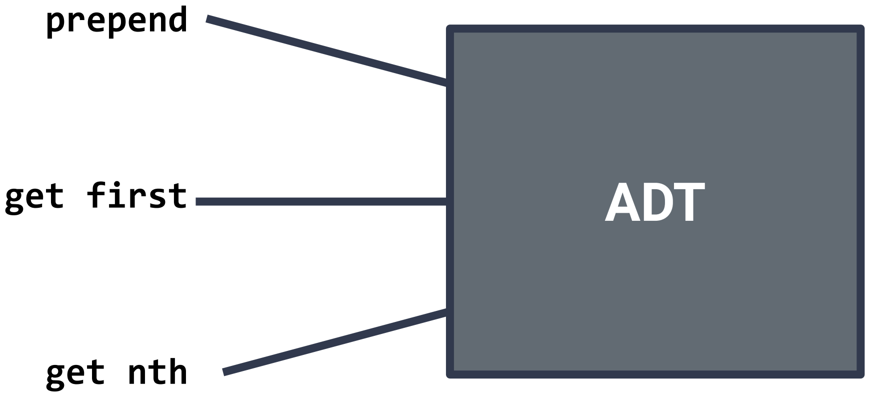
Thought Experiment

An Abstract Data Type is a specification of **what** a data structure can do



Thought Experiment

Often, many data structures can satisfy a given ADT...how do you choose?



Thought Experiment

Data Structure 1
- Very fast `prepend`, `get first`
- Very slow `get nth`

Data Structure 2
- Very fast `get nth`, `get first`
- Very slow `prepend`

Data Structure 3
- Very fast `get nth`, `get first`
- Occasionally slow `prepend`

Which is better?

Thought Experiment

Data Structure 1 (LinkedList)
- Very fast **`prepend`**, **`get first`**
- Very slow **`get nth`**

Data Structure 2 (Array)
- Very fast **`get nth`**, **`get first`**
- Very slow **`prepend`**

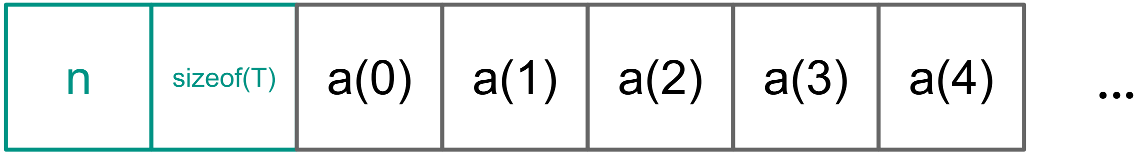
Data Structure 3 (ArrayList…in reverse)
- Very fast **`get nth`**, **`get first`**
- Occasionally slow **`prepend`**

Which is better?

IT DEPENDS!

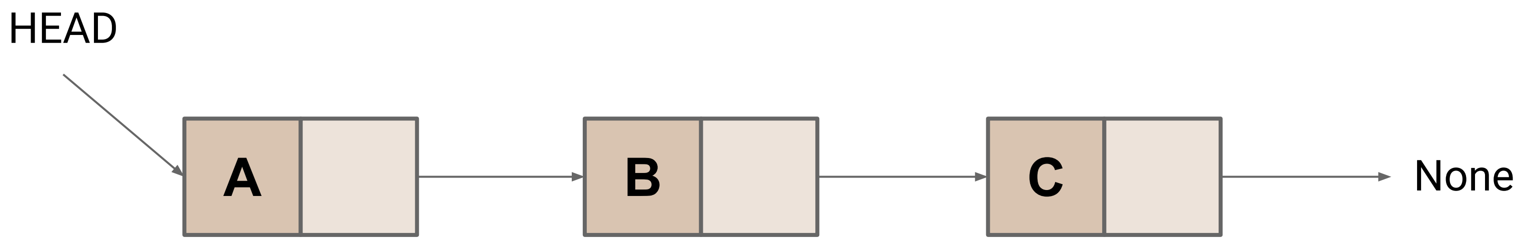
A (very) Brief Refresher: Array

- An array is an ordered container (elements stored one after another)
- Array elements are all stored in a contiguous block of memory

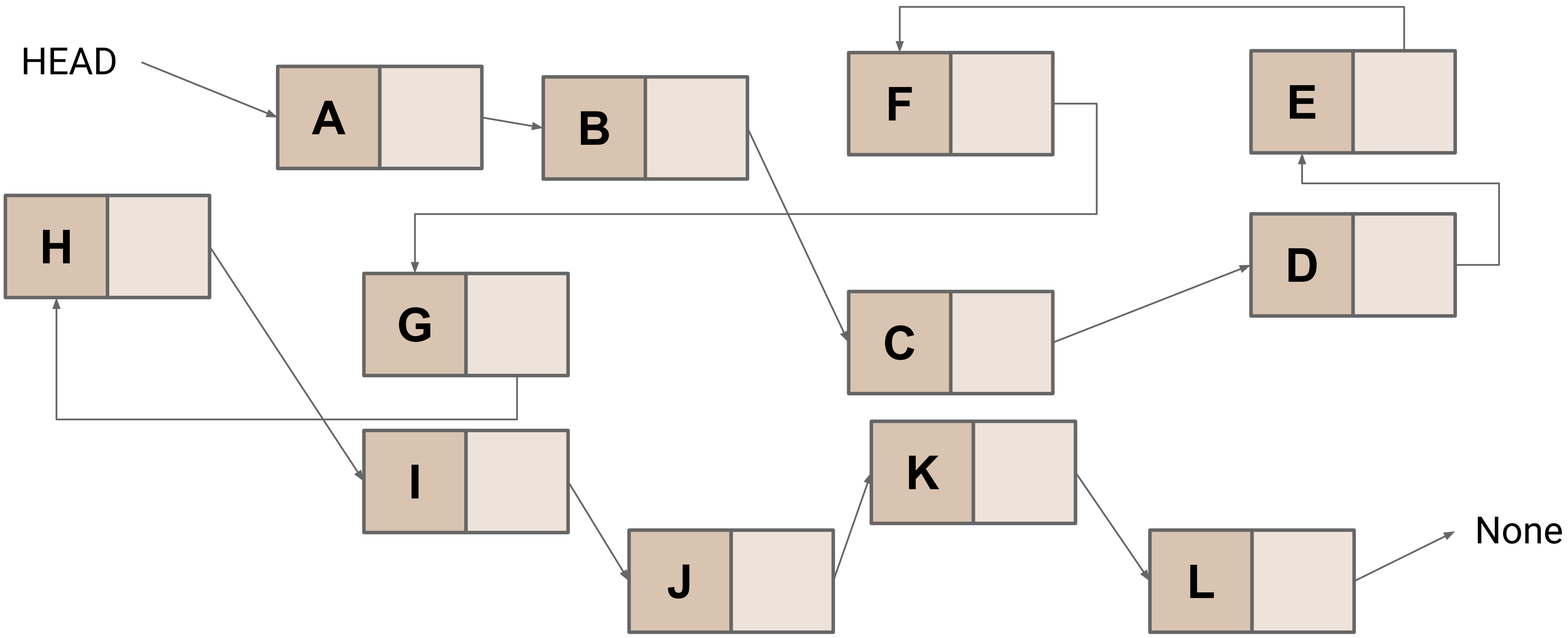


A (very) Brief Refresher: Linked Lists

- Also an ordered container
- Each element stores a pointer to the next element
  - …not necessarily in a contiguous block of memory

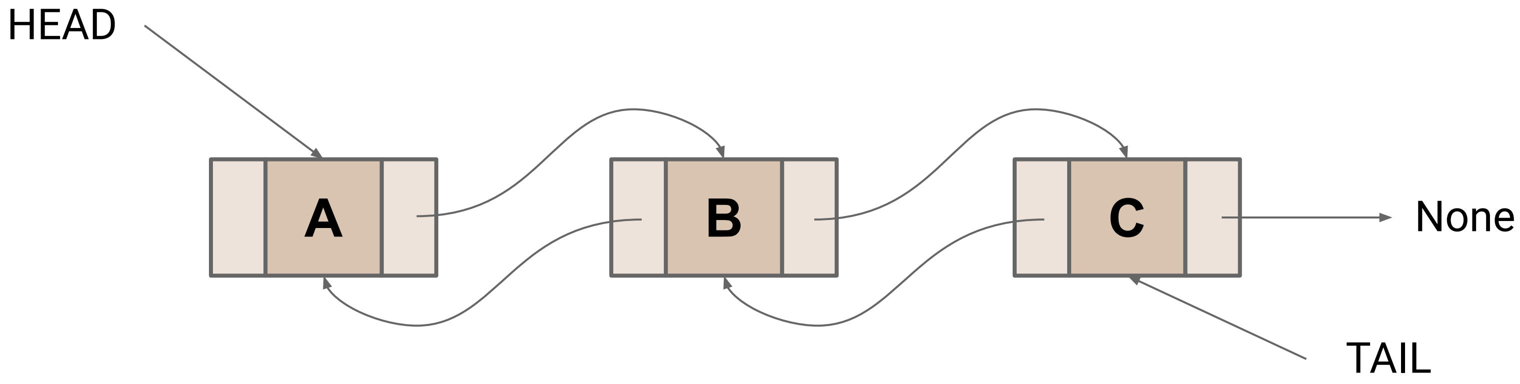


A (very) Brief Refresher: Linked Lists



A (very) Brief Refresher: Linked Lists

- Can also be doubly linked (a next AND a prev pointer per node)
- PA1 will have you implementing a **Sorted Doubly Linked List** with some minor twists



Thought Experiment

Data Structure 1 (LinkedList)

- Very **fast** `prepend`, `get first`
- Very **slow** `get nth`

Data Structure 2 (Array)

- Very **fast** `get nth`, `get first`
- Very **slow** `prepend`

Data Structure 3 (ArrayList…in reverse)

- Very **fast** `get nth`, `get first`
- Occasionally **slow** `prepend`

What is "fast"? "slow"?

Attempt #1: Wall-clock time?

- What is fast?
  - 10s? 100ms? 10ns?
  - …it depends on the task
- Algorithm vs Implementation
  - Compare Grace Hopper’s implementation to yours
- What machine are you running on?
  - Your old laptop? A lab machine? The newest, shiniest processor on the market?
- What bottlenecks exist? CPU vs IO vs Memory vs Network…

Attempt #1: Wall-clock time?

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Wall-clock time is not terribly useful…

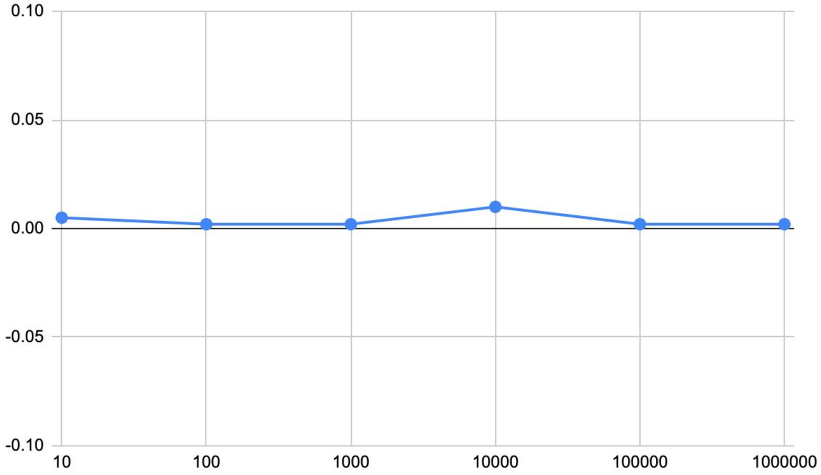
Analysis Checklist

1. Don’t think in terms of wall-time, think in terms of “number of steps”

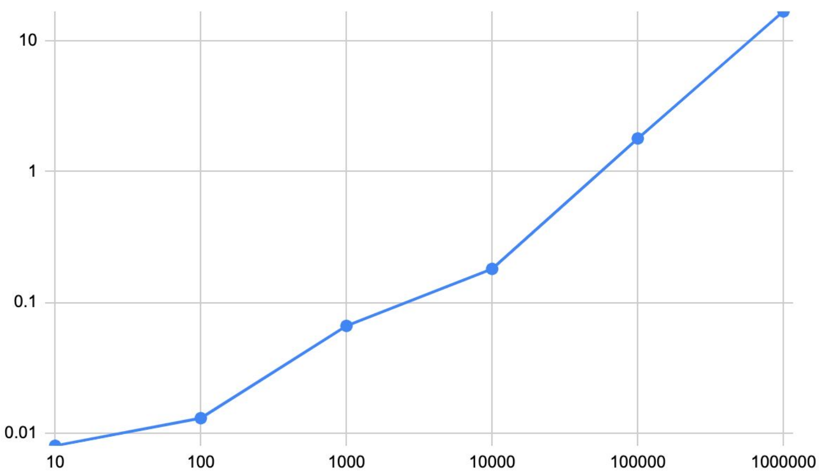
Let’s do a quick demo…

Comparing Random Access for Array vs List

Array



List



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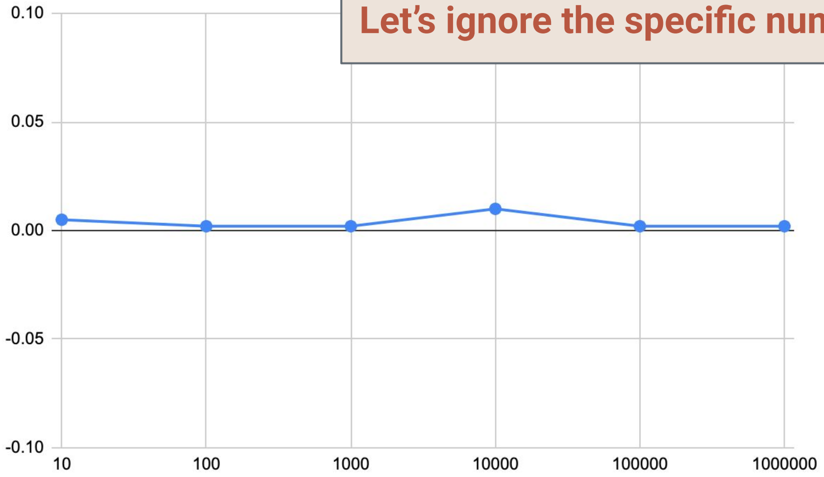
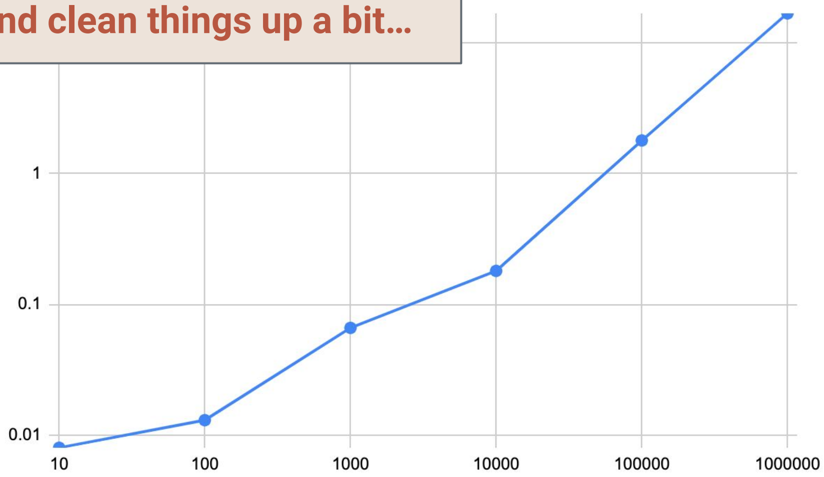


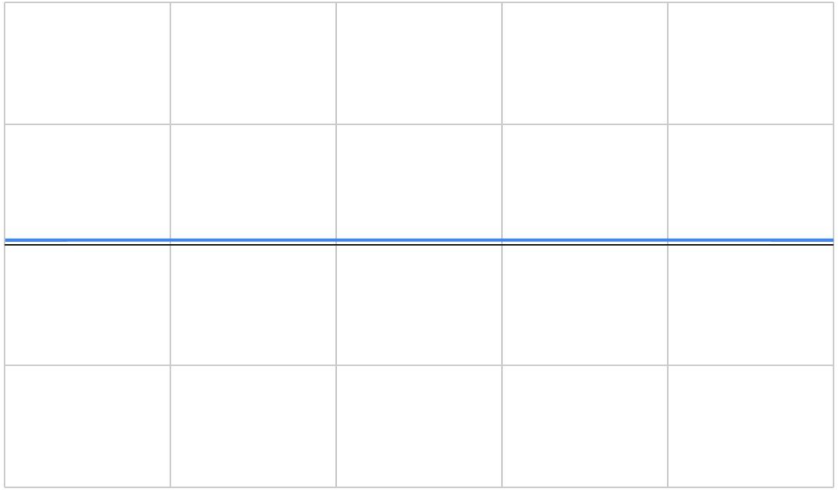
List



Let's ignore the specific numbers and clean things up a bit…

Comparing Random Access for Array vs List

Array



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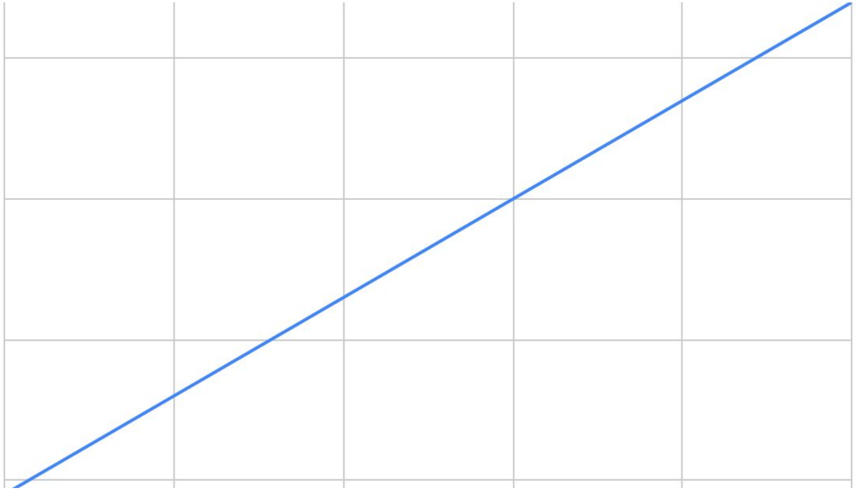

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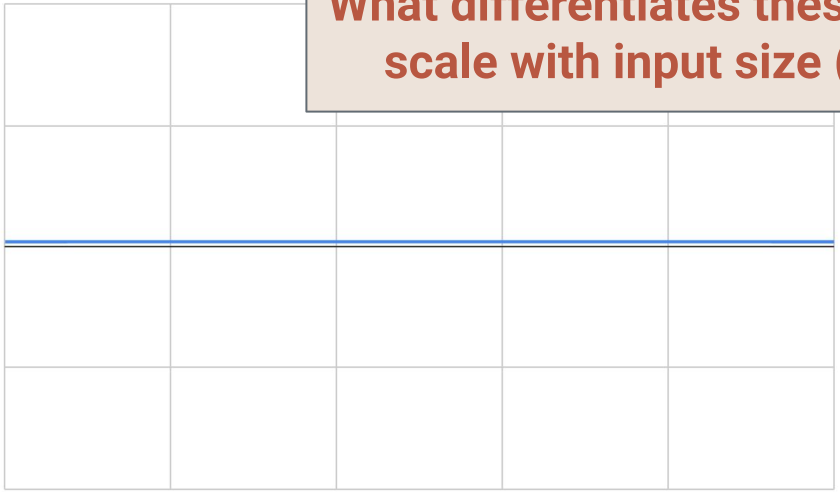

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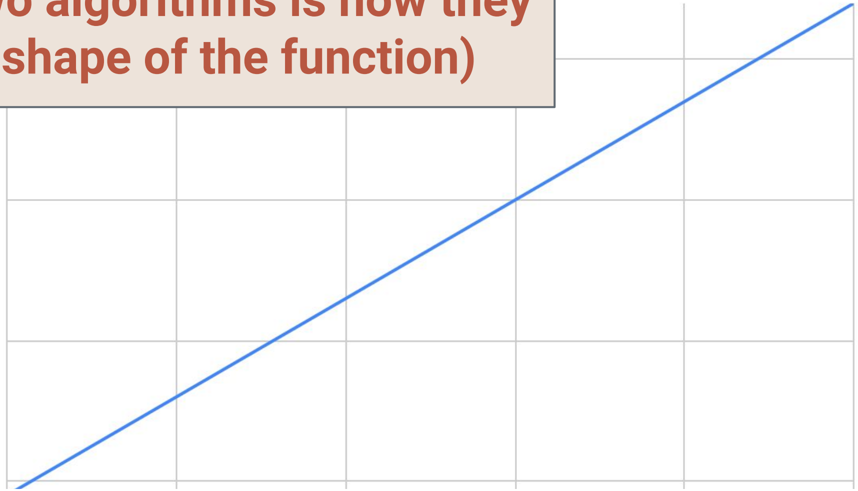

What differentiates these two algorithms is how they scale with input size (the shape of the function)

Analysis Checklist

1. Don’t think in terms of wall-time, think in terms of “number of steps”
2. **To give a useful solution, we should take “scale” into account**
    - **How does the runtime change as we change the size of the input?**

Counting Steps

```
public void updateUsers(User[] users) {
  x = 1;
  for(user : users) {
    user.id = x;
    x = x + 1;
  }
}
```

Counting Steps

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public void updateUsers(User[] users) {
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1

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$$1 + \sum_{user \in users}$$

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$$1 + \sum_{user \in users} 4$$

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Counting Steps

```
public void userFullName(User[] users, int id) {
  User user = users[id];
  String fullName = user.firstName + user.lastName;
  return fullName;
}
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Counting Steps

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7 steps…(sort of, more details later)

Counting Steps

```
public void totalReads(User[] users, Post[] posts) {
  int totalReads = 0;
  for(post : posts) {
    int userReads = 0;
    for(user : users) {
      if(user.readPost(post)){ userReads += 1; }
    }
    totalReads += userReads;
  }
}
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Counting Steps

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1 ·

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$$1 + \sum_{post \in posts}$$

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$$1 + \sum_{post \in posts} \Bigg(3$$

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$$1 + \sum_{post \in posts} \left(3 + \sum_{user \in users} \right.$$

Counting Steps

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  }
}
```

$$1 + \sum_{post \in posts} \left(3 + \sum_{user \in users} 2 \right)$$

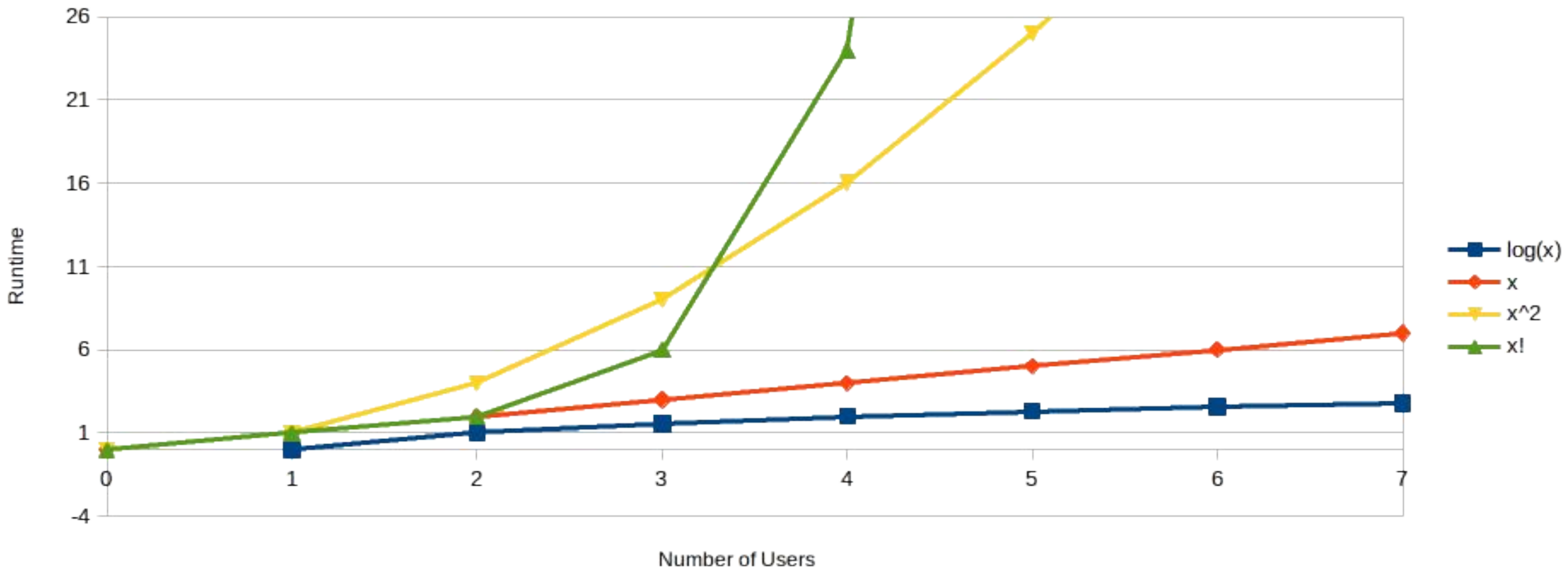
Steps to "Functions"

Now that we have number of steps in terms of summations…

…which we can simplify (like in WA1) into mathematical functions…

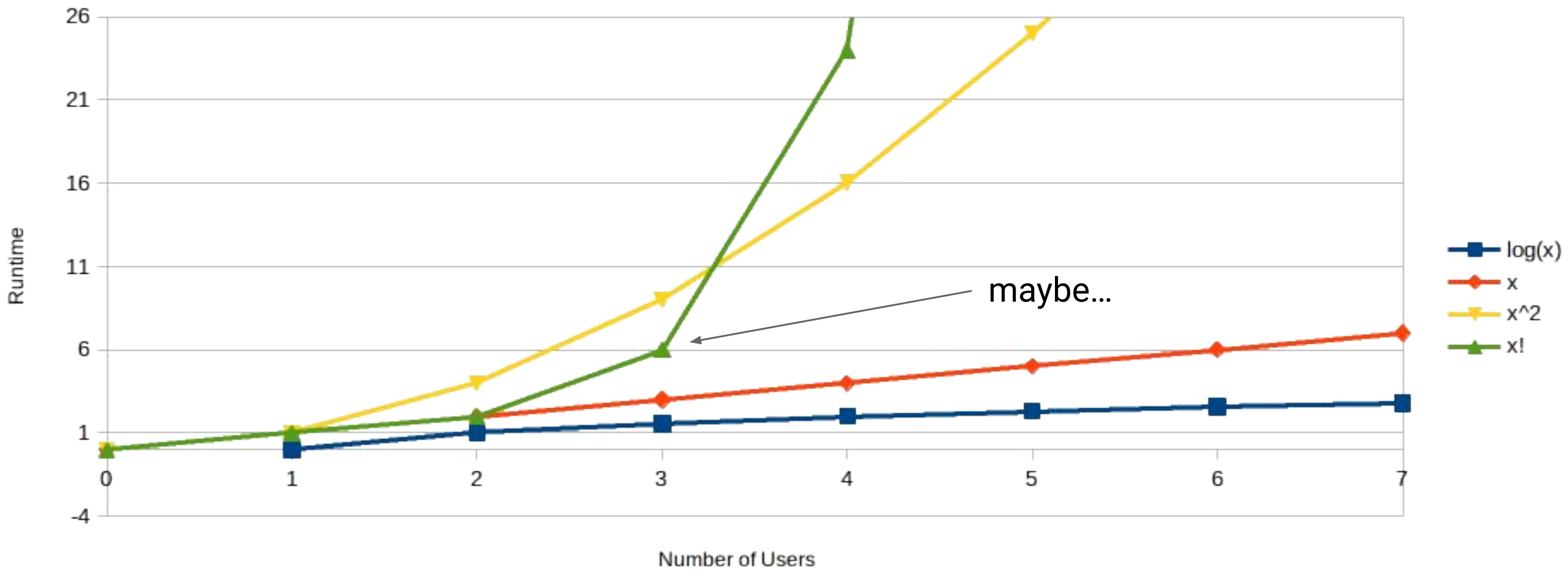
We can start analyzing runtime as a function

Runtime as a Function



Would you consider an algorithm that takes |Users|! number of steps?

Runtime as a Function



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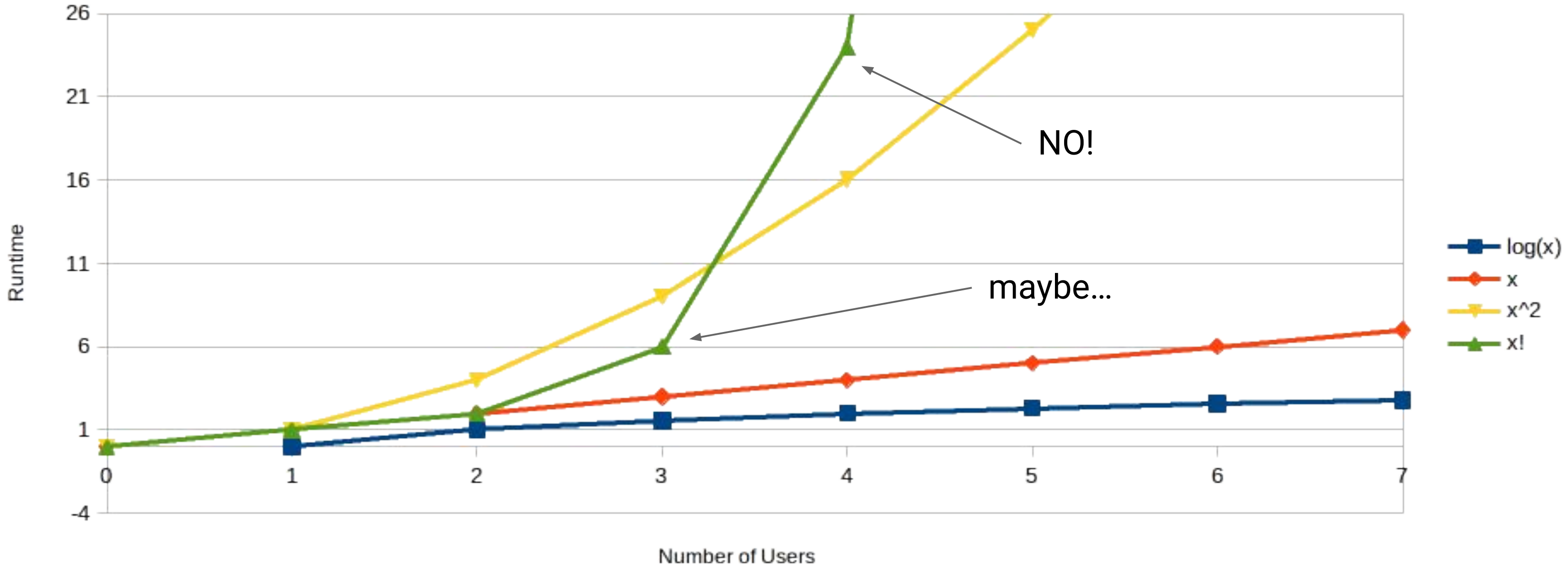

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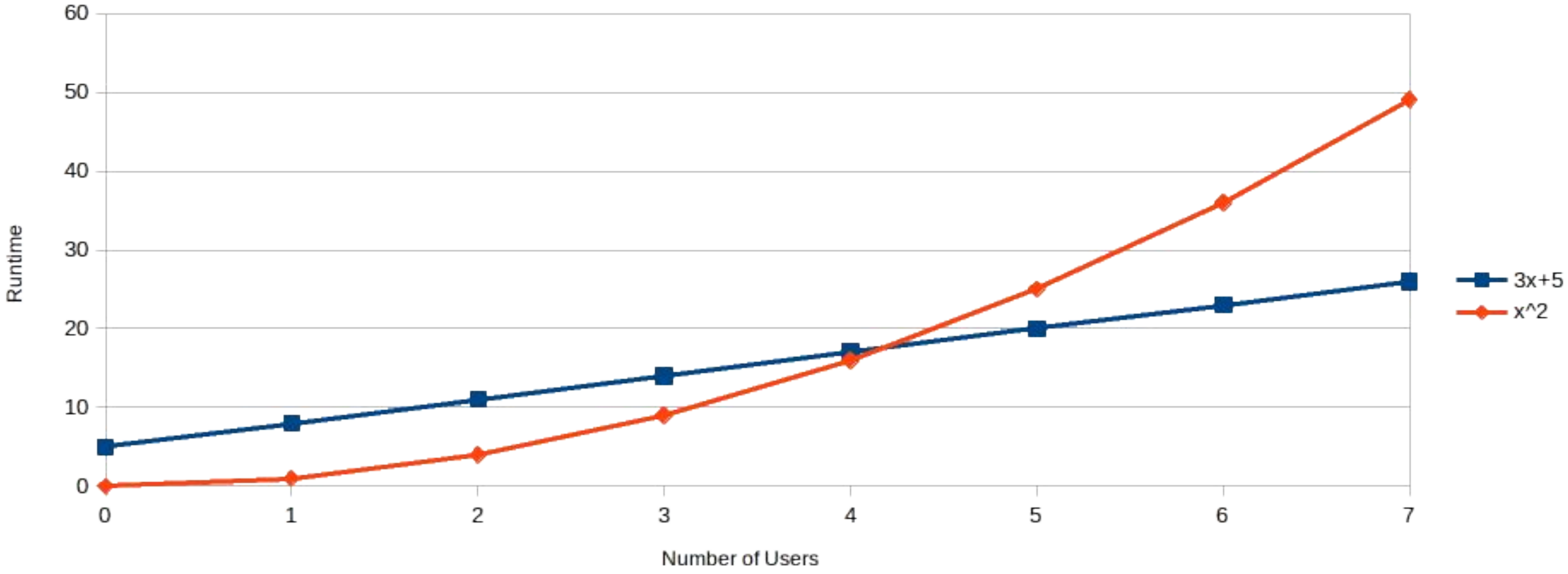

Which is better? 3x|Users|+5 or $|Users|^2$

Analysis Checklist

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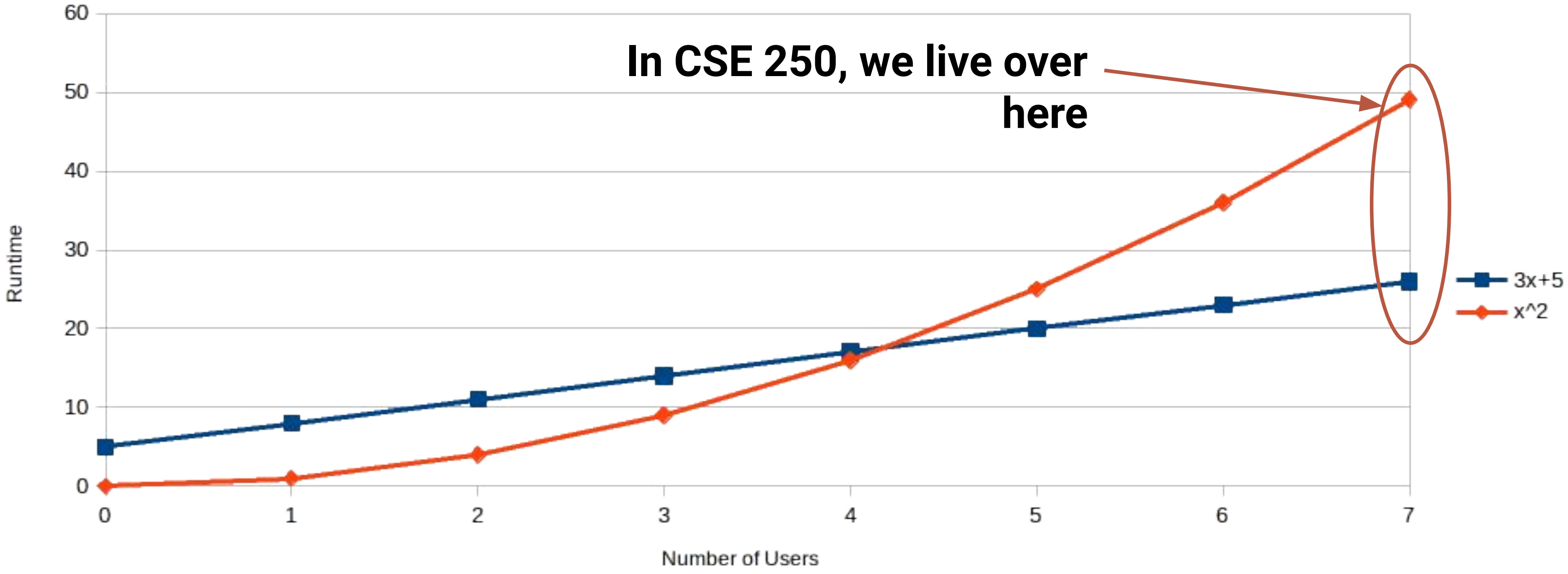
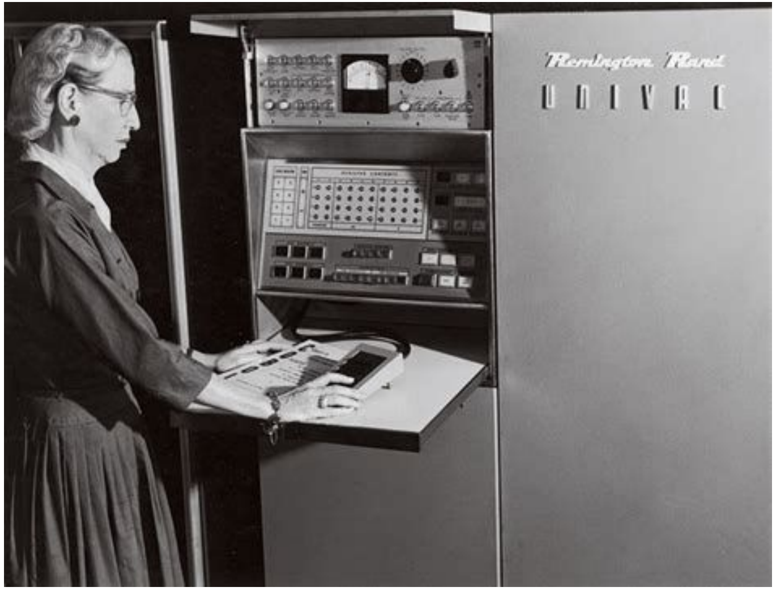
    - How does the runtime change as we change the size of the input?
3. **Focus on “large” inputs**
    - **Rank functions based on how they behave at large scales**

Runtime as a Function



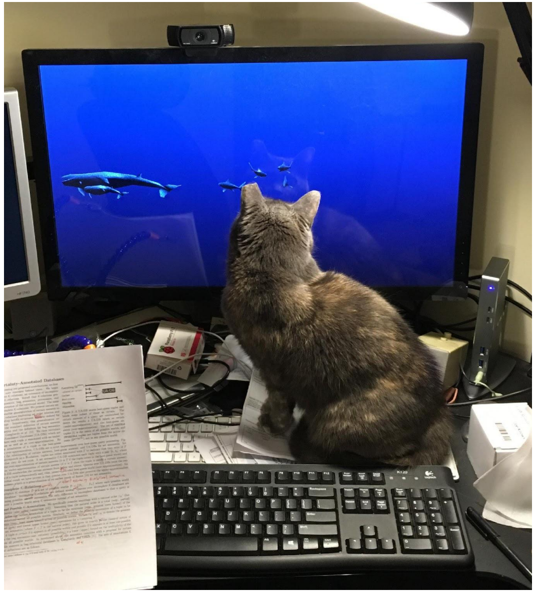
Which is better? 3x|Users|+5 or |Users|^2

Goal: Ignore implementation details



Seasoned Pro Implementation

vs

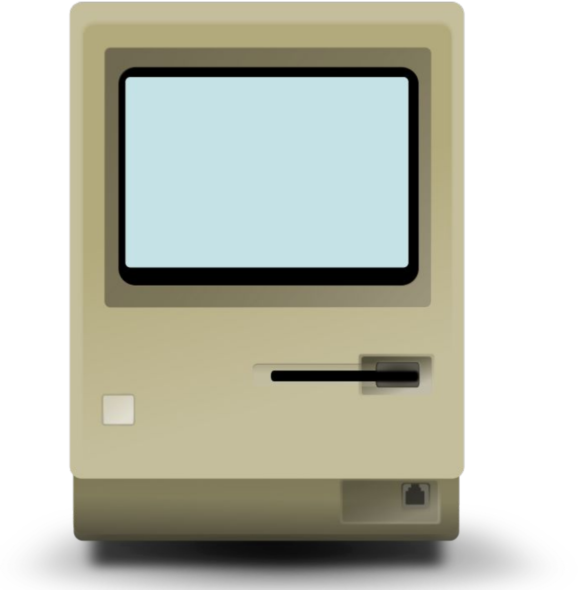


Error 23: Cat on Keyboard

Goal: Ignore execution environment

Intel i9

vs



Motorola 68000

Goal: Judge the Algorithm Itself

- How fast is a step? Don’t care
  - Only count number of steps
- Can this be done in two steps instead of one?
  - “3 steps per user” vs “some number of steps per user”
  - Sometimes we don’t care…sometimes we do

Analysis Checklist

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   - How does the runtime change as we change the size of the input?
3. Focus on "large" inputs
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4. **Decouple algorithm from infrastructure/implementation**
   - **Asymptotic notation…?**