CSE 250
Data Structures

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Lec 14: Midterm #1 Review

Midterm Procedure

- Exam is during normal class time. Same time, same place.
- Seating is assigned randomly
  - Wait outside the room until instructed to enter
  - Immediately place all bags/electronics at the front of the room
- At your seat you should have:
  - Writing utensil
  - UB ID card
  - One 8.5x11 cheatsheet (front and back) if desired
  - Summation/Log rules will be provided

Content Overview

	Analysis Tools/Techniques	ADTs	Data Structures
Week 2/3	Asymptotic Analysis, (Unqualified) Runtime Bounds		
Week 3		Sequence	Array, LinkedList
Week 4	Amortized Runtime	List	ArrayList, LinkedList
Week 5	Induction, Expected Runtime	Stack/Queue	ArrayList, LinkedList

Analysis Tools and Techniques

Recap of Runtime Complexity

Big-Θ — Tight Bound

- Growth functions are in the **same** complexity class
- If f(n) ∈ **Θ**(g(n)) then an algorithm taking f(n) steps is as "exactly" as fast as one that takes g(n) steps.

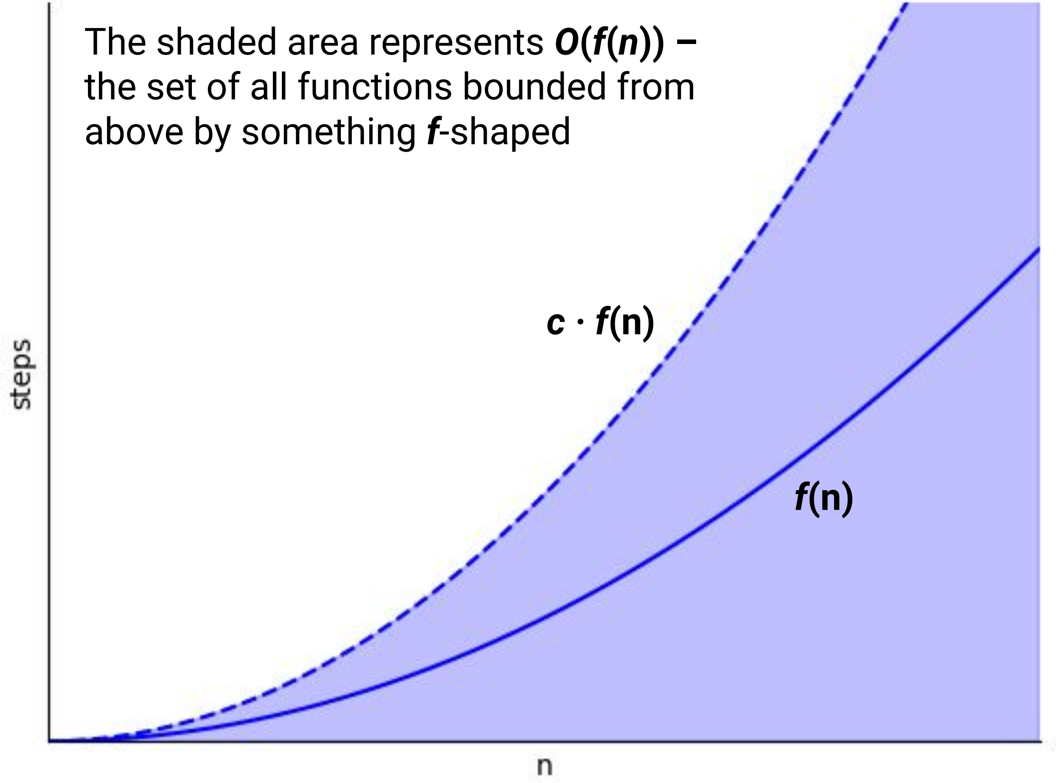
Big-O — Upper Bound

- Growth functions in the **same or smaller** complexity class.
- If f(n) ∈ O(g(n)), then an algorithm that takes f(n) steps is *at least as fast* as one taking g(n) (but it may be even faster).

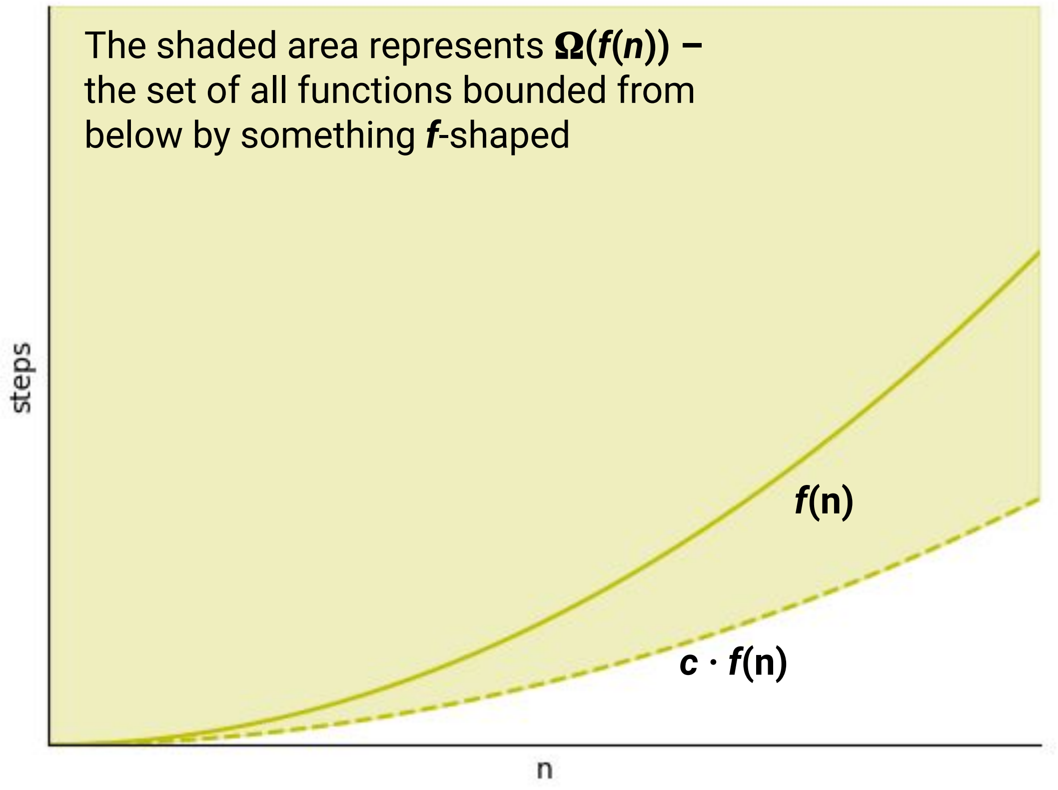
Big-Ω — Lower Bound

- Growth functions in the **same or bigger** complexity class
- If f(n) ∈ **Ω**(g(n)), then an algorithm that takes f(n) steps is *at least as slow* as one that takes g(n) steps (but it may be even slower)

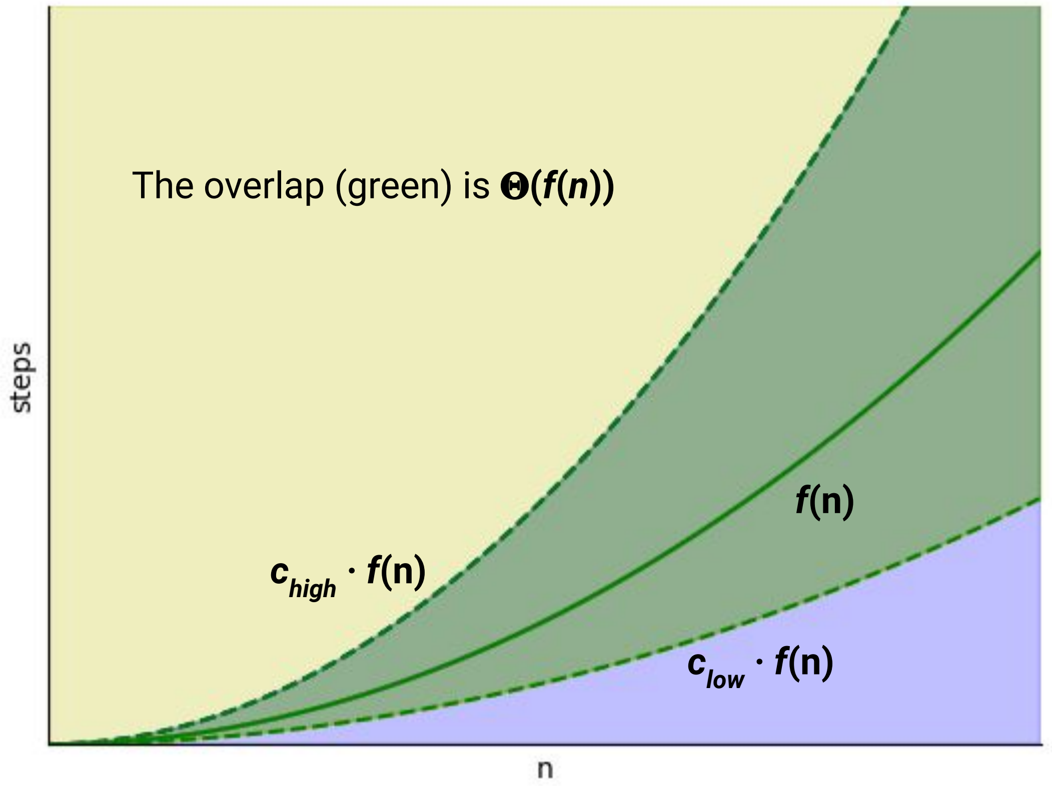
Bounded from Above: Big O



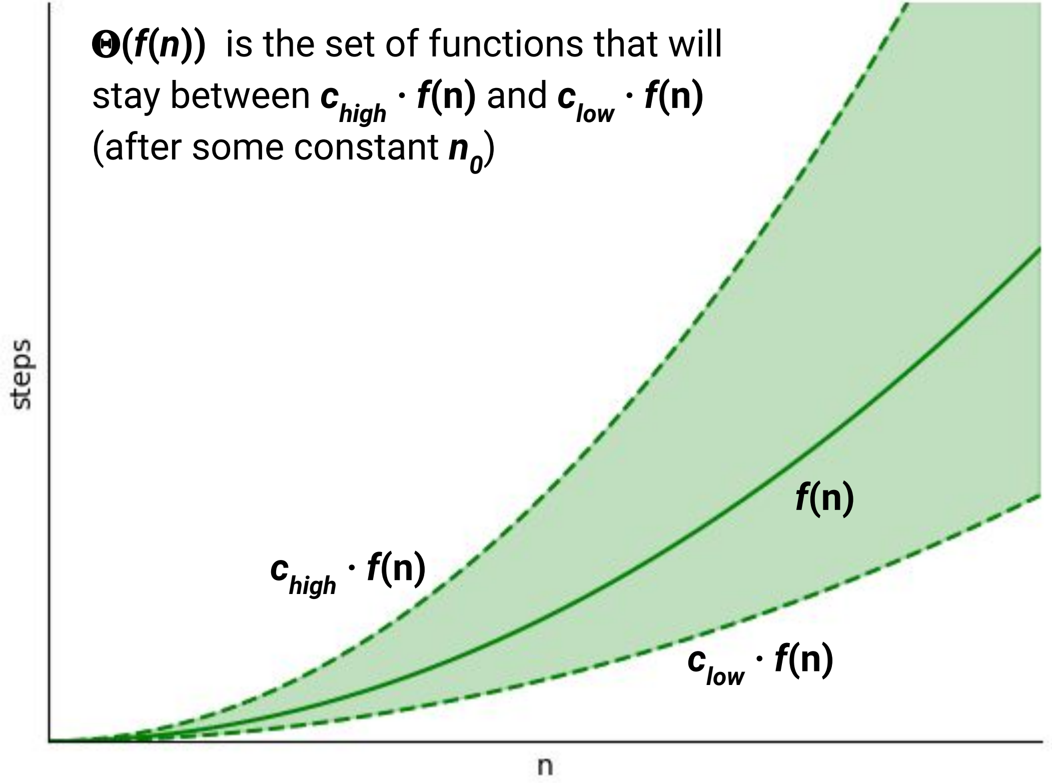
Bounded from Below: Big Ω



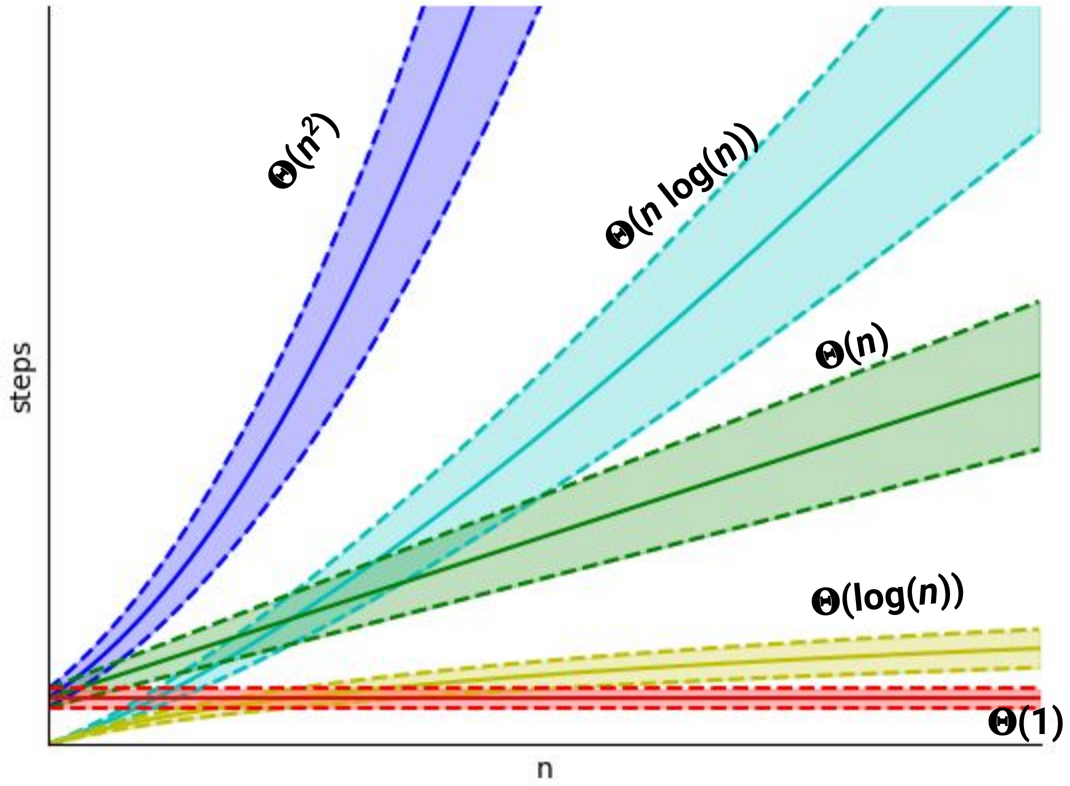
Complexity Class: Big Θ



Complexity Class: Big Θ



Complexity Class Ranking



$\Theta(1) < \Theta(\log(n)) < \Theta(n) < \Theta(n \log(n)) < \Theta(n^2) < \Theta(n^3) < \Theta(2^n)$

Common Runtimes (in order of complexity)

Constant Time:	**$\Theta(1)$**
Logarithmic Time:	**$\Theta(\log(n))$**
Linear Time:	**$\Theta(n)$**
Quadratic Time:	**$\Theta(n^2)$**
Polynomial Time:	**$\Theta(n^k)$ for some $k > 0$**
Exponential Time:	**$\Theta(c^n)$ (for some $c \geq 1$)**

Formal Definitions

$\mathbf{f(n)} \in \boldsymbol{O}\mathbf{(g(n))}$ iff exists some constants $\mathbf{c}$, $\mathbf{n_0}$ s.t.

$\mathbf{f(n) \leq c * g(n)}$ for all $\mathbf{n > n_0}$

$\mathbf{f(n)} \in \boldsymbol{\Omega}\mathbf{(g(n))}$ iff exists some constants $\mathbf{c}$, $\mathbf{n_0}$ s.t.

$\mathbf{f(n) \geq c * g(n)}$ for all $\mathbf{n > n_0}$

$\mathbf{f(n)} \in \boldsymbol{\Theta}\mathbf{(g(n))}$ iff $\mathbf{f(n)} \in \boldsymbol{O}\mathbf{(g(n))}$ and $\mathbf{f(n)} \in \boldsymbol{\Omega}\mathbf{(g(n))}$

Shortcut

What complexity class do each of the following belong to:

$\boldsymbol{f(n) = 4n + n^2 \in \Theta(n^2)}$

$\boldsymbol{g(n) = 2^n + 4n \in \Theta(2^n)}$

$\boldsymbol{h(n) = 100\, n \log(n) + 73n \in \Theta(n \log(n))}$

Shortcut: Just consider the complexity of the most dominant term

Multi-class Functions

$$T(n) = \begin{cases} n^2 & \text{if } n \text{ is even} \\ n & \text{if } n \text{ is odd} \end{cases}$$

It is not bounded from above by $\boldsymbol{n}$, therefore it cannot be in $\boldsymbol{\Theta(n)}$

It is not bounded from below by $\boldsymbol{n^2}$, therefore it cannot be in $\boldsymbol{\Theta(n^2)}$

What is the tight upper bound of this function? $\boldsymbol{T(n) \in O(n^2)}$

What is the tight lower bound of this function? $\boldsymbol{T(n) \in \Omega(n)}$

What is the complexity class of this function? It does not have one!

Amortized Runtime

If n calls to a function take $\Theta(f(n))$...

We say the **Amortized Runtime** is $\Theta(f(n)\ /\ n)$

The **amortized runtime** of `add` on an `ArrayList` is: $\Theta(n/n) = \Theta(1)$

The **unqualified runtime** of `add` on an `ArrayList` is: $O(n)$

Expected Runtime

If our algorithm involves some sort of random process, we can still analyze the runtime as a growth function **T(*n*)**...

But we can also analyze the <u>**expected runtime**</u>, **E[T(*n*)]**

Example: $T_{quicksort}(n) \in O(n^2)$ and $E[T_{quicksort}(n)] \in O(n \log(n))$

What guarantees do you get?

If $f(n)$ is a Tight Bound

The algorithm always runs in $cf(n)$ steps

← Unqualified runtime

If $f(n)$ is a Worst-Case Bound

The algorithm always runs in at most $cf(n)$

If $f(n)$ is an Amortized Worst-Case Bound

n invocations of the algorithm **always** run in $cnf(n)$ steps

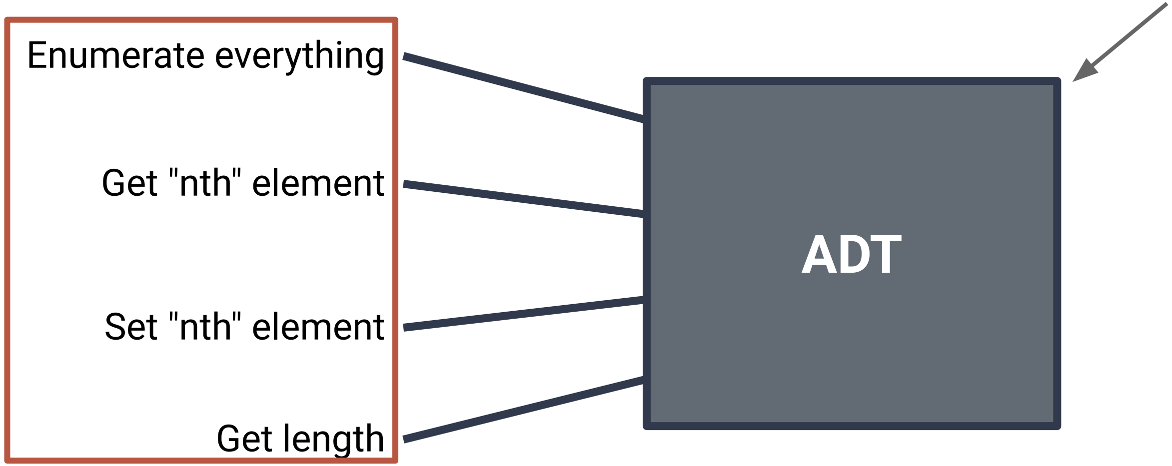
If $f(n)$ is an Average Bound

...we don't have any guarantees

ADTs and Data Structures

Abstract Data Types (ADTs)

The specification of **what** a data structure can do



What's in the box? …we don't know, and in some sense…we don't care

Usage is governed by **what** we can do, not **how** it is done

Abstract Data Type vs Data Structure

ADT

The interface to a data structure

*Defines **what** the data structure can do*

Many data structures can implement the same ADT

Data Structure

The implementation of one (or more) ADTs

*Defines **how** the different tasks are carried out*

Different data structures will excel at different tasks

Abstract Data Type vs Data Structure

ADT

The interface to

*Defines **what** the*

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Data Structure

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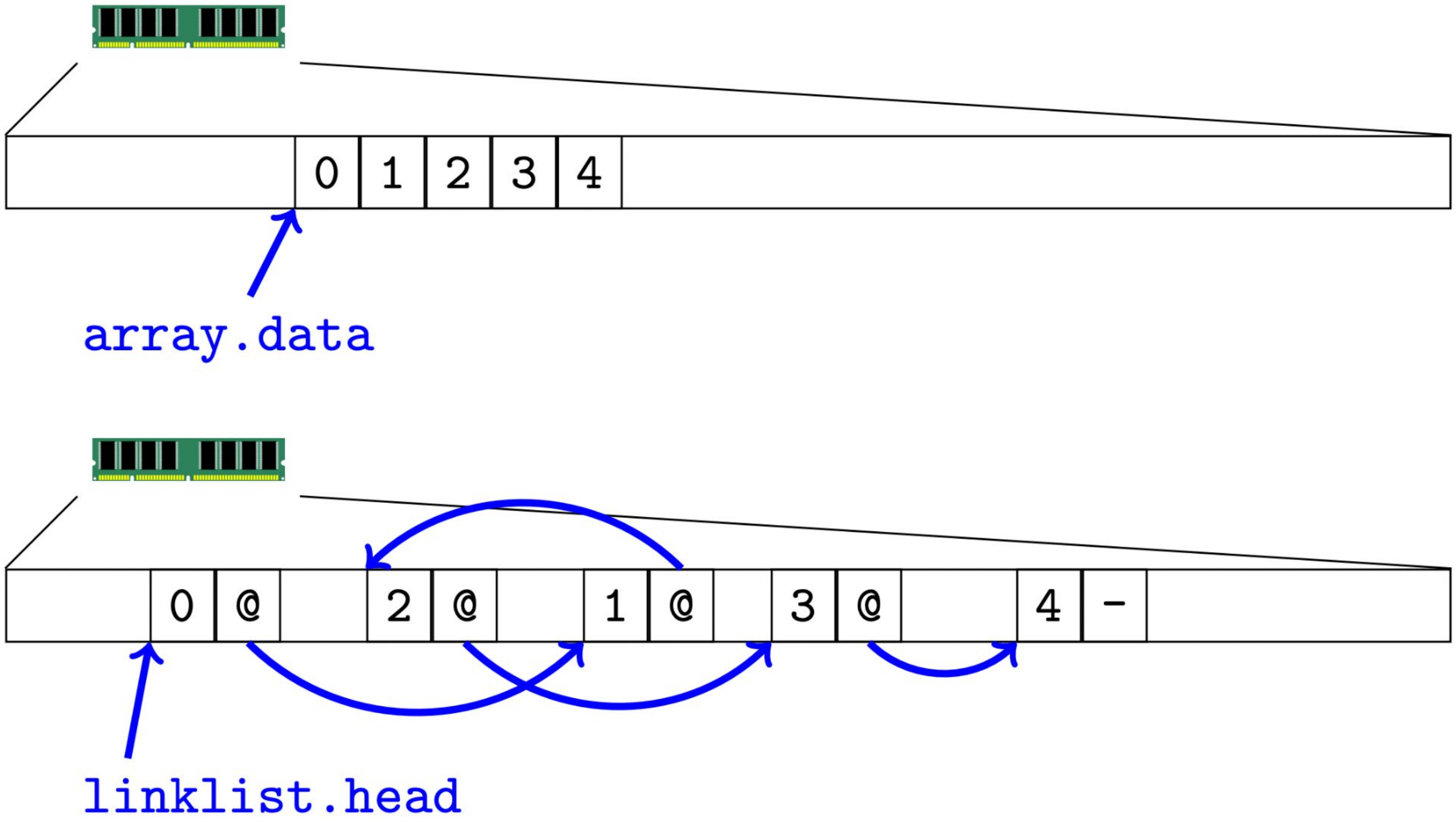
Think about the Linked List we implemented for PA1.

The internal structure and the mental model of our sequence are very different.

The Sequence ADT

```
public interface Sequence<E> {
  public E get(idx: Int);
  public void set(idx: Int, E value);
  public int size();
  public Iterator<E> iterator();
}
```



Arrays and Linked Lists in Memory

The List ADT

```
public interface List<E>
    extends Sequence<E> { // Everything a sequence has, and...
  /** Extend the sequence with a new element at the end */
  public void add(E value);

  /** Extend the sequence by inserting a new element */
  public void add(int idx, E value);

  /** Remove the element at a given index */
  public void remove(int idx);
}
```

Runtime Summary

	ArrayList	Linked List (by index)	Linked List (by reference)
`get(...)`	$\Theta(1)$	$\Theta(\text{idx})$ or $O(n)$	$\Theta(1)$
`set(...)`	$\Theta(1)$	$\Theta(\text{idx})$ or $O(n)$	$\Theta(1)$
`size()`	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$
`add(...)`	$O(n)$, Amortized $\Theta(1)$	$\Theta(\text{idx})$ or $O(n)$	$\Theta(1)$
`remove(...)`	$O(n)$	$\Theta(\text{idx})$ or $O(n)$	$\Theta(1)$

Runtime Summary

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`get(...)`	$\Theta(1)$	$\Theta(\text{idx})$ or $O(n)$	$\Theta(1)$
`set(...)`			
`size()`			
`add(...)`	$O(n)$, A		
`remove(...)`	$O(n)$	$\Theta(\text{idx})$ or $O(n)$	$\Theta(1)$

Also consider how we can search by value… ie searching for a value in our SortedList from PA1, searching an unsorted Array vs searching a sorted Array, etc…

Stacks

Represents a stack of objects on top of one another

```java
public class Stack<E> {

  public void push(E value); // Add value to the "top" of the stack

  public E pop();    // Remove and return the top of the stack

  public E peek();  // Return the top of the stack

}
```

Queues

Outside of the US, "queueing" is lining up, ie at Starbucks

```
public class Queue<E> {

  public void add(E value); // Add value to the "back" of the queue

  public E remove();    // Remove and return the front of the queue

  public E peek();  // Return the front of the queue

}
```

Recap

Stacks: Last In First Out (LIFO)

- Push (put item on top of the stack) $\Theta(1)$ (or amortized $O(1)$)
- Pop (take item off top of stack) $\Theta(1)$
- Peek (peek at top of stack) $\Theta(1)$

Queues: First in First Out (FIFO)

- Enqueue (put item on the end of the queue) $\Theta(1)$ (or amortized $O(1)$)
- Dequeue (take item off the front of the queue) $\Theta(1)$
- Peek (peek at the item in the front of the queue) $\Theta(1)$