CSE 250
Data Structures

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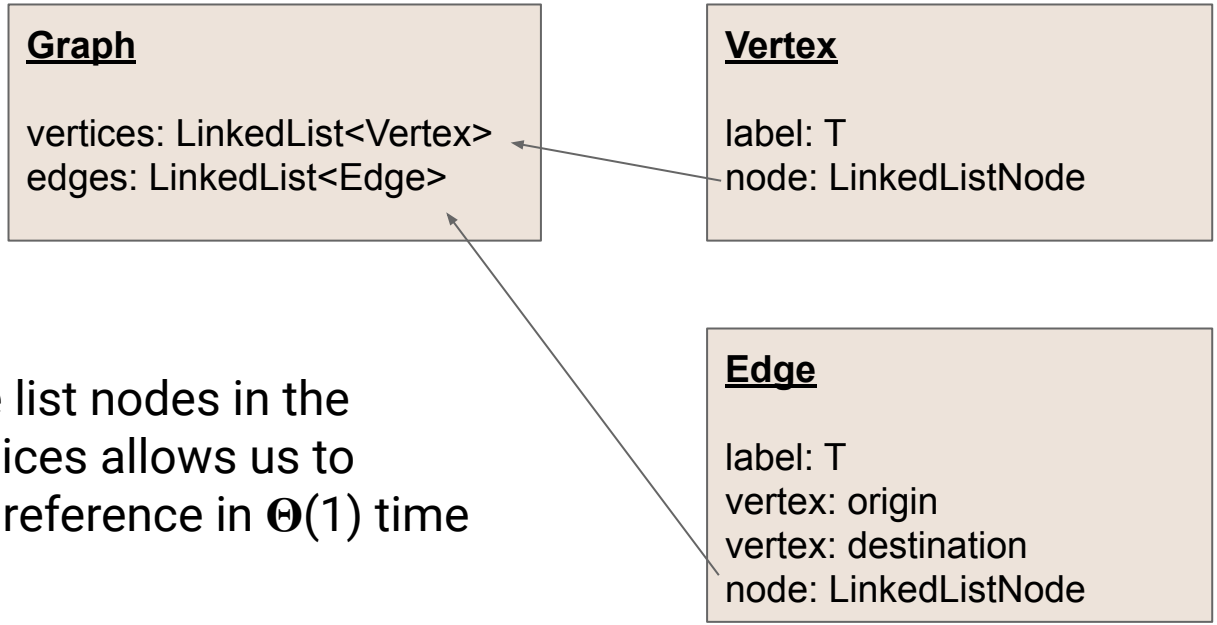
Lec 19: Adjacency Lists and Matrices

Announcements

- WA3 due Sunday

Edge List Summary



Storing the list nodes in the edges/vertices allows us to remove by reference in $\Theta(1)$ time

Edge List Summary

- `addEdge, addVertex`: $O(1)$
- `removeEdge`: $O(1)$
- `removeVertex`: $O(m)$
- `vertex.incidentEdges`: $O(m)$
- `vertex.edgeTo`: $O(m)$
- **Space Used: $O(n) + O(m)$**

Edge List Summary

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← Involves checking every edge in the graph

How can we improve?

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Idea: Store the in/out edges for each vertex!

(Called an adjacency list)

Adjacency List

```
1  public class Vertex<V,E> {
2    public Node<Vertex> node;
3    public List<Edge> inEdges = new CustomLinkedList<Edge>();
4    public List<Edge> outEdges = new CustomLinkedList<Edge>();
5    /*...*/
6  }
```

Each vertex stores a list of **`inEdges`** and **`outEdges`**, which are maintained as the graph is modified…

What functions need to change to maintain these lists?

Adjacency List

```
1  public Edge addEdge(Vertex orig, Vertex dest, E label) {
2    Edge e = new Edge(orig, dest, label);
3    e.node = edges.add(e);
4    orig.outEdges.add(e);
5    dest.inEdges.add(e);
6    return e;
7  }
```

← When we add an edge to the graph, also add it to the appropriate adjacency lists

What is the complexity of **addEdge** now?

Adjacency List

```
1  public Edge addEdge(Vertex orig, Vertex dest, E label) {
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7  }
```

← When we add an edge to the graph, also add it to the appropriate adjacency lists

What is the complexity of **addEdge** now? Still $\Theta(1)$

Adjacency List

```
1  public void removeEdge(Edge edge) {
2    edges.remove(edge.node);
3    edge.orig.outEdges.remove(edge);
4    edge.dest.inEdges.remove(edge);
5  }
```

← When we remove an edge from the graph, also remove it from the adjacency lists

What is the complexity of **`removeEdge`** now?

Adjacency List

```
1  public void removeEdge(Edge edge) {
2    edges.remove(edge.node);
3    edge.orig.outEdges.remove(edge);
4    edge.dest.inEdges.remove(edge);
5  }
```

← When we remove an edge from the graph, also remove it from the adjacency lists

What is the complexity of **`removeEdge`** now? ***O*(deg(orig) + deg(dest)) :(**

But how can we fix this?

Adjacency List

```
1  public class Edge<V,E> {
2    public Node<Edge> node;
3    public Node<Edge> inNode;
4    public Node<Edge> outNode;
5    /*...*/
6  }
```

Each Edge now also stores a reference to the nodes in each adjacency list

Adjacency List

```
1  public Edge addEdge(Vertex orig, Vertex dest, E label) {
2    Edge e = new Edge(orig, dest, label);
3    e.node = edges.add(e);
4    e.outNode = orig.outEdges.add(e);
5    e.inNode = dest.inEdges.add(e);
6    return e;
7  }
```

← When we add an edge to the graph, also add it to the appropriate adjacency lists AND store the node refs in the Edge object

What is the complexity of **addEdge** now? Still $\Theta(1)$

Adjacency List

```
1  public void removeEdge(Edge edge) {
2    edges.remove(edge.node);
3    edge.orig.outEdges.remove(edge.outNode);
4    edge.dest.inEdges.remove(edge.inNode);
5  }
```

← When we remove an edge from the graph, also remove it from the adjacency lists (remove by reference)

What is the complexity of **`removeEdge`** now?

Adjacency List

```
1  public void removeEdge(Edge edge) {
2    edges.remove(edge.node);
3    edge.orig.outEdges.remove(edge.outNode);
4    edge.dest.inEdges.remove(edge.inNode);
5  }
```

← When we remove an edge from the graph, also remove it from the adjacency lists (remove by reference)

What is the complexity of **`removeEdge`** now? $\Theta(1)$

Adjacency List

So, we are able to store and maintain adjacency lists in each vertex while still keeping a $\Theta(1)$ runtime for **`addVertex`**, **`addEdge`**, and **`removeEdge`**

How much extra space is used?

Adjacency List

So, we are able to store and maintain adjacency lists in each vertex while still keeping a $\Theta(1)$ runtime for **`addVertex`**, **`addEdge`**, and **`removeEdge`**

How much extra space is used? $\Theta(1)$ per edge

Each edge only appears in 3 lists:

- The edge list
- One vertices inList
- One vertices outList

Adjacency List

So, we are able to store and maintain adjacency lists in each vertex while still keeping a $\Theta(1)$ runtime for **`addVertex`**, **`addEdge`**, and **`removeEdge`**

How much extra space is used? $\Theta(1)$ per edge

Each edge only appears in 3 lists:

- The edge list
- One vertices inList
- One vertices outList

But now what have we gained?

Adjacency List

```
1  public void removeVertex(Vertex v) {
2    for(edge : v.getIncidentEdges()) {
3      removeEdge(edge.node)
4    }
5    vertices.remove(v.node);
6  }
```

What is the complexity of **`removeVertex`** now?

Adjacency List

```
1  public void removeVertex(Vertex v) {
2    for(edge : v.getIncidentEdges()) {
3      Θ(1)
4    }
5    Θ(1)
6  }
```

What is the complexity of **`removeVertex`** now?

Adjacency List

```
1  public void removeVertex(Vertex v) {
2    for(edge : v.getIncidentEdges()) {
3      Θ(1)
4    }
5    Θ(1)
6  }
```

We now have a reference to the list of edges in $\Theta(1)$ time, and there are **deg(v)** edge in the list

What is the complexity of **`removeVertex`** now?

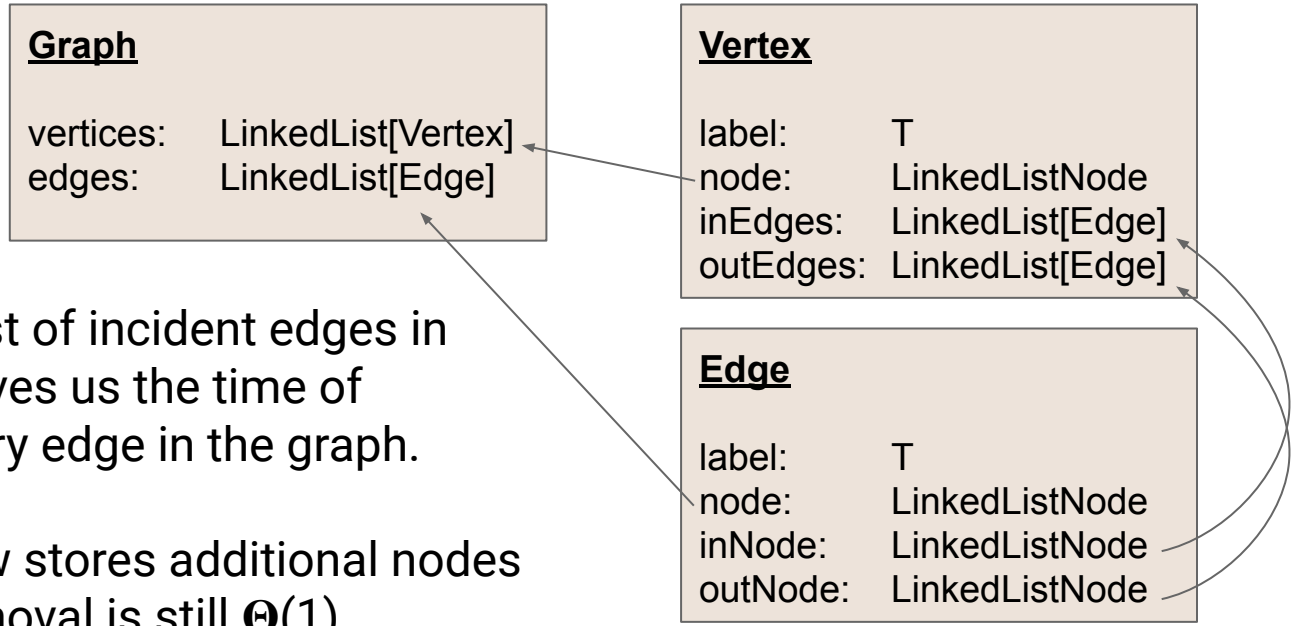
Adjacency List

```
1  public void removeVertex(Vertex v) {
2    for(edge : v.getIncidentEdges()) {
3      Θ(1)
4    }
5    Θ(1)
6  }
```

We now have a reference to the list of edges in $\Theta(1)$ time, and there are **deg(v)** edge in the list

What is the complexity of **`removeVertex`** now? $\Theta(deg(v))$

Adjacency List Summary



Storing the list of incident edges in the vertex saves us the time of checking every edge in the graph.

The edge now stores additional nodes to ensure removal is still $\Theta(1)$

Adjacency List Summary

- `addEdge, addVertex`: $\Theta(1)$
- `removeEdge`: $\Theta(1)$
- `removeVertex`: $\Theta(\deg(\text{vertex}))$
- `vertex.incidentEdges`: $\Theta(\deg(\text{vertex}))$
- `vertex.edgeTo`: $\Theta(\deg(\text{vertex}))$
- Space Used: $\Theta(n) + \Theta(m)$

Adjacency List Summary

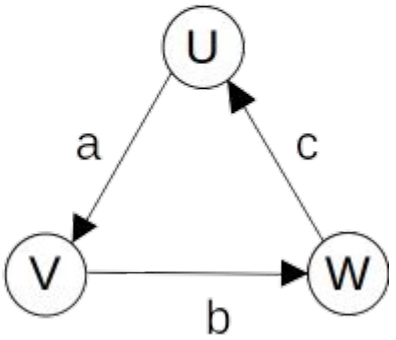
- `addEdge, addVertex`: **Θ(1)**
- `removeEdge`: **Θ(1)**
- `removeVertex`: **Θ(deg(vertex))**
- `vertex.incidentEdges`: **Θ(deg(vertex))**
- `vertex.edgeTo`: **Θ(deg(vertex))**
- **Space Used: Θ(*n*) + Θ(*m*)**

Now we already know what edges are incident without having to check them all

Adjacency Matrix

Destination

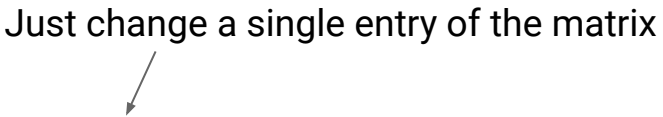
Origin	U	V	W
U	-	***a***	-
V	-	-	***b***
W	***c***	-	-



Adjacency Matrix Summary

- `addEdge, removeEdge`:
- `addVertex, removeVertex`:
- `vertex.incidentEdges`:
- `vertex.edgeTo`:
- **Space Used:**

Adjacency Matrix Summary



- `addEdge, removeEdge`: $\Theta(1)$
- `addVertex, removeVertex`:
- `vertex.incidentEdges`:
- `vertex.edgeTo`:
- **Space Used:**

Adjacency Matrix Summary

- `addEdge, removeEdge`: $\Theta(1)$
- `addVertex, removeVertex`: $\Theta(n^2)$
- `vertex.incidentEdges`:
- `vertex.edgeTo`:
- **Space Used:**

Resize and copy the whole matrix

Adjacency Matrix Summary

- `addEdge, removeEdge`: $\Theta(1)$
- `addVertex, removeVertex`: $\Theta(n^2)$
- `vertex.incidentEdges`: $\Theta(n)$
- `vertex.edgeTo`:
- **Space Used:**

Check the row and column for that vertex

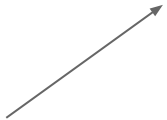
Adjacency Matrix Summary

- `addEdge, removeEdge`: $\Theta(1)$
- `addVertex, removeVertex`: $\Theta(n^2)$
- `vertex.incidentEdges`: $\Theta(n)$
- `vertex.edgeTo`: $\Theta(1)$
- **Space Used:**

Check a single entry of the matrix

Adjacency Matrix Summary

- `addEdge, removeEdge`: $\Theta(1)$
- `addVertex, removeVertex`: $\Theta(n^2)$
- `vertex.incidentEdges`: $\Theta(n)$
- `vertex.edgeTo`: $\Theta(1)$
- **Space Used: $\Theta(n^2)$**



How does this relate to space of edge/adjacency lists?

Adjacency Matrix Summary

- `addEdge, removeEdge`: $\Theta(1)$
- `addVertex, removeVertex`: $\Theta(n^2)$
- `vertex.incidentEdges`: $\Theta(n)$
- `vertex.edgeTo`: $\Theta(1)$
- **Space Used: $\Theta(n^2)$**

How does this relate to space of edge/adjacency lists? **If the matrix is "dense" it's about the same**

So…what do we do with our graphs?
…next lecture