# CSE 250 Recitation

4/1-4/2: Binary Trees and Heaps

# Binary Trees

- A binary tree is a special type of tree where each node can only have up to two children
- Even though this might seem like a simple condition for a tree to meet, we can use this condition to create many different types of binary trees
- In this course, we will focus on two main types of binary trees
  - Heaps
  - Search Trees

# Heaps

- A heap is a partially ordered complete binary tree
  - A complete tree is a tree where every leaf is in the two deepest layers and leaves are added from left to right
  - A heap is partially ordered because you can only create an order between parents and children, but not between siblings
- A heap needs to fulfill a special condition in order to be correct
  - In a max heap, every parent must be greater than its children
  - In a min heap, every parent must be less than its children

# Heaps

- Since a heap is a complete tree, it can be stored in an array in O(n) space
  - An array is helpful because we can find each child in O(1) time
  - To find the children of the node at index *i* we can use the following formula
    - Index of left child: 2(*i*)+1
    - Index of right child: 2(*i*)+2
- The heapify algorithm can be used to convert any array into a heap

# Heapify

Trace the execution of Heapify on the following array

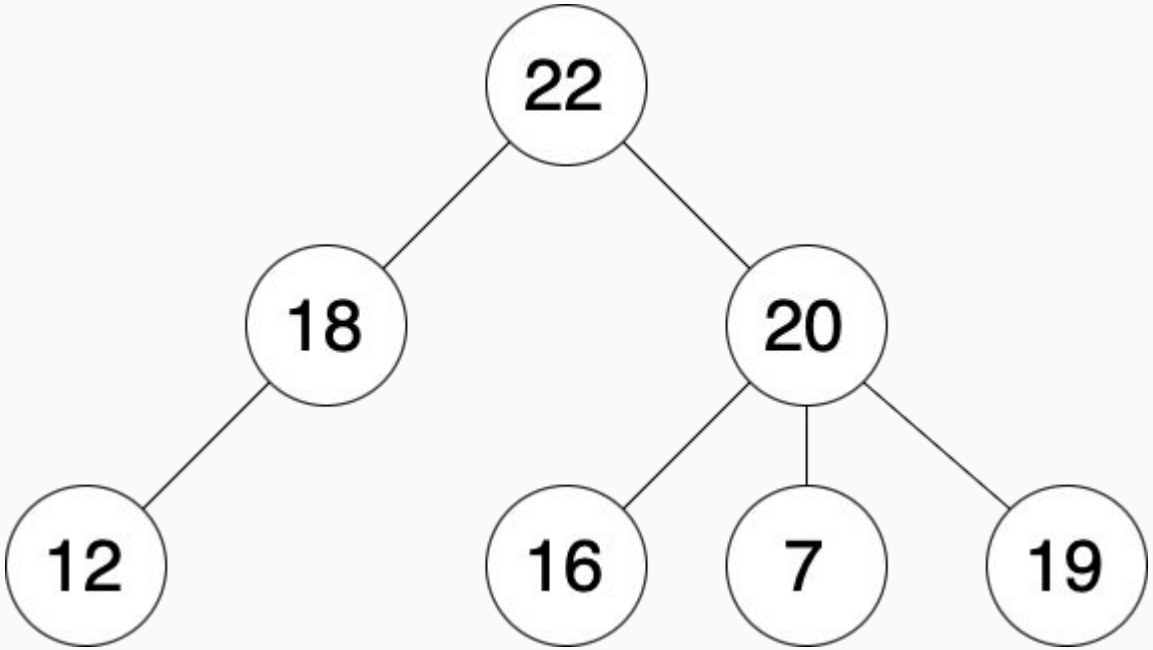
9 6 8 1 5 4 15 3 7 14 11 10 2 13 12

# Binary Search Trees

- A binary search tree is a tree where:
  - Every node in the right subtree of a parent node is greater than the parent node
  - Every node in the left subtree of a parent node is less than the parent node
- These two conditions mean that each parent partitions the binary search tree into a lesser subtree and greater subtree

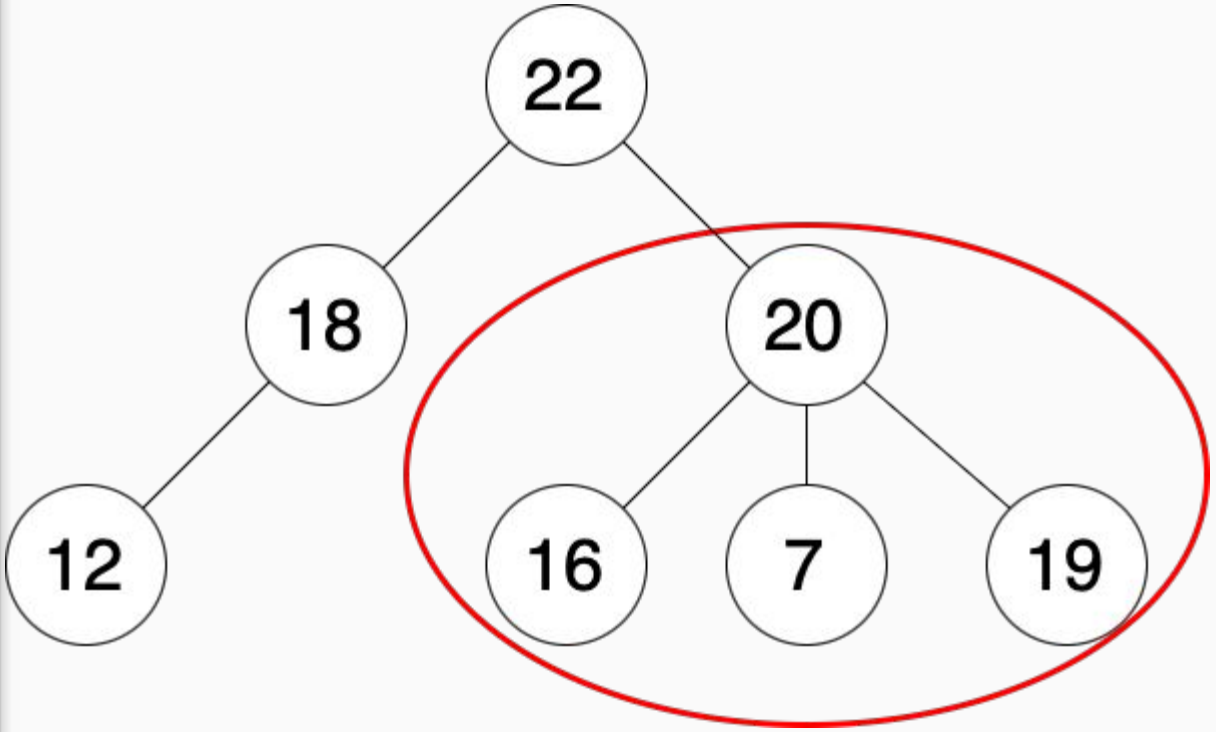
# Binary Tree Examples

- Is this tree a binary tree?
- Is this tree a heap, binary search tree, or both?



# Binary Tree Examples

- Is this tree a binary tree?
  - No because node 20 has three children
- Is this tree a heap, binary search tree, or both?
  - This tree is neither a heap or BST because it is not a binary tree



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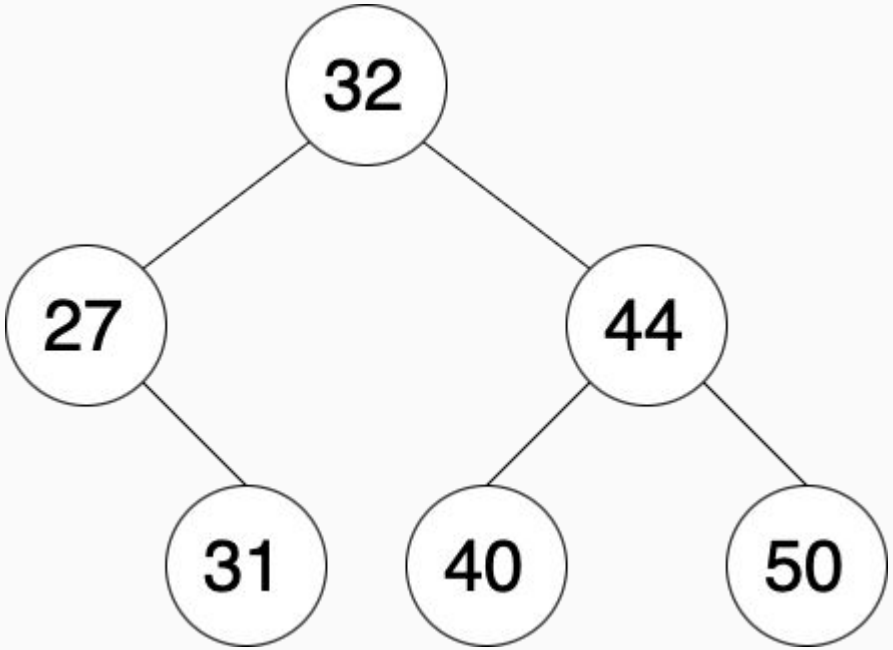
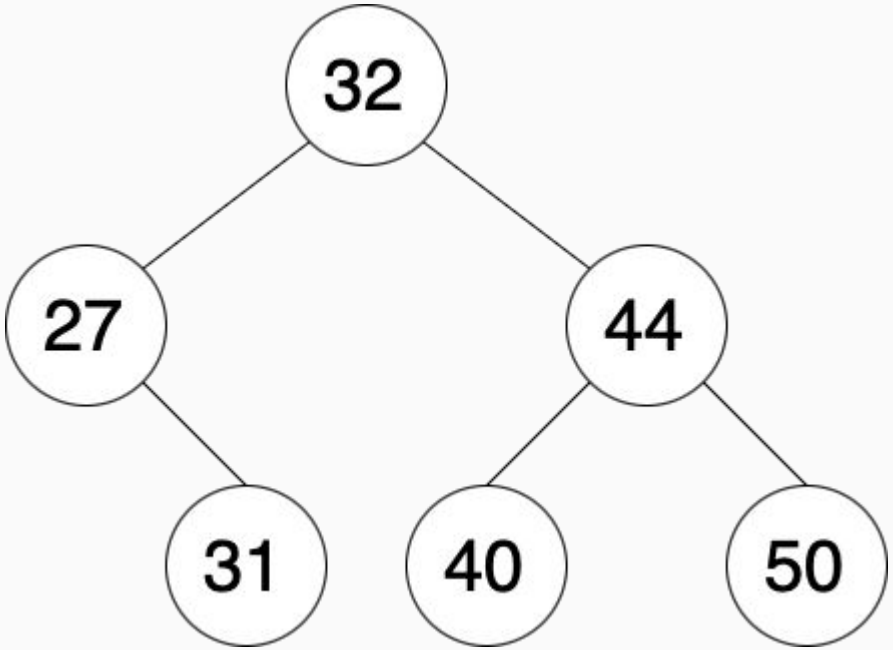


# Binary Tree Examples

- Is this tree a binary tree?
  - Yes, every node has at most two children
- Is this tree a heap, binary search tree, or both?
  - This tree is a BST because every parent properly partitions the tree
  - This tree is not a heap because it's not a complete tree (27 needs a left child before a right child). It also doesn't have the proper ordering.



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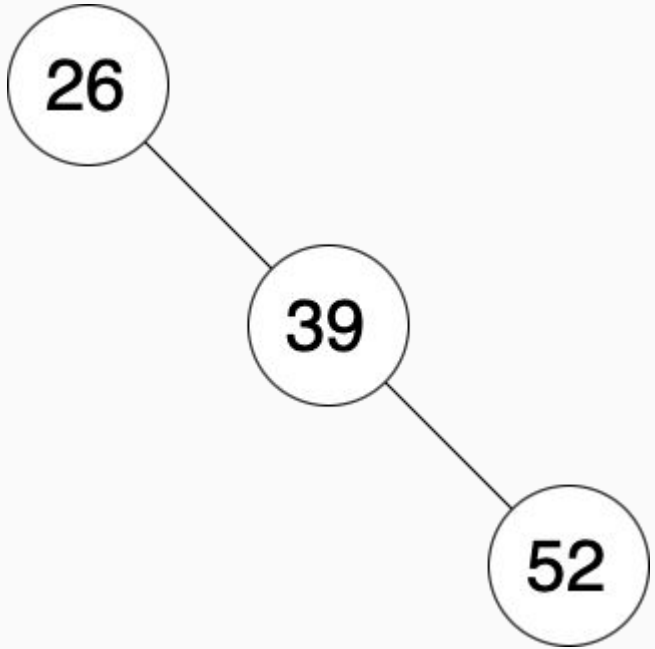
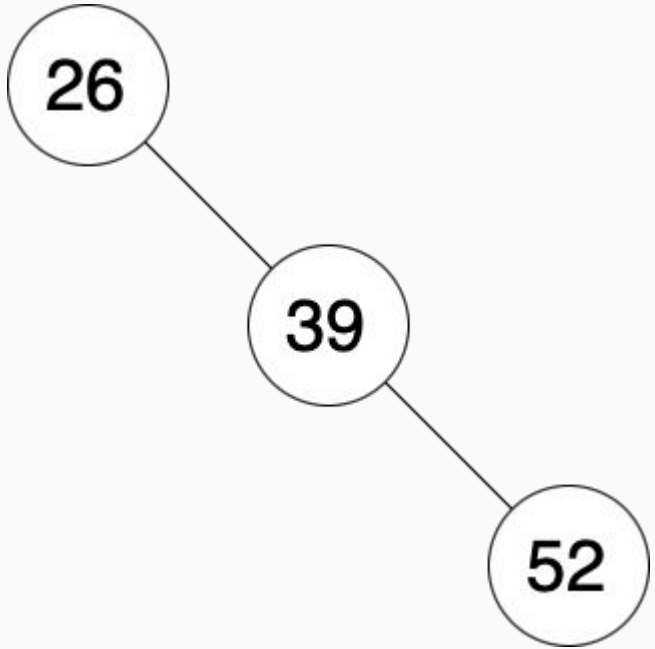


# Binary Tree Examples

- Is this tree a binary tree?
  - Yes, no node has more than two children
- Is this tree a heap, binary search tree, or both?
  - This tree is a BST because all the right children are greater than their parents
  - This tree is not a heap because it's not a complete tree



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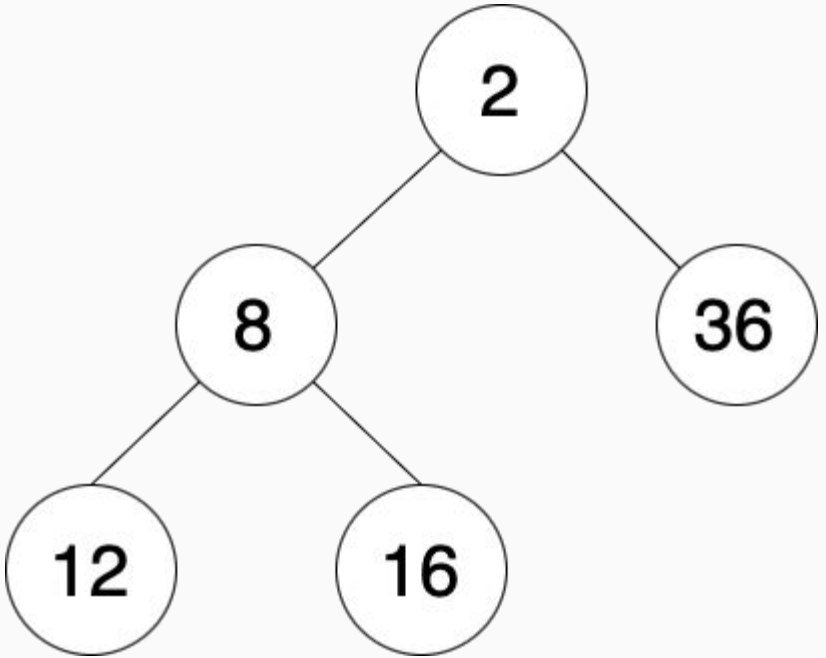
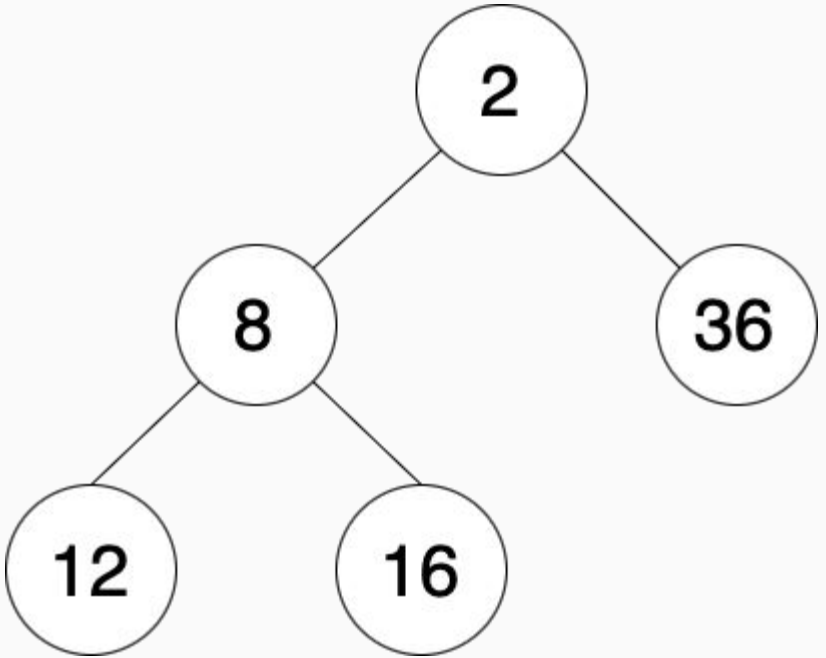


# Binary Tree Examples

- Is this tree a binary tree?
  - Yes, every node has at most two children
- Is this tree a heap, binary search tree, or both?
  - This tree is a min heap because it's a complete tree and every parent is less than its children
  - This tree is not a BST because every left child is greater than its parent



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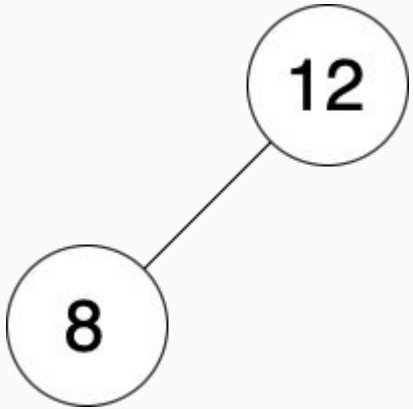
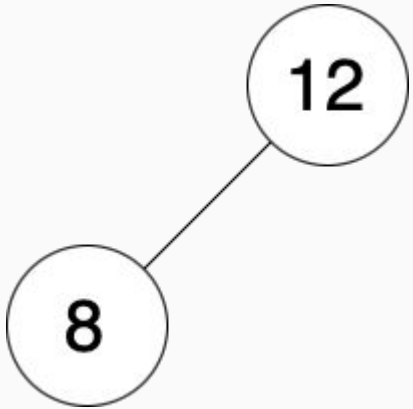


# Binary Tree Examples

- Is this tree a binary tree?
  - Yes, no node has more than two children
- Is this tree a heap, binary search tree, or both?
  - This tree is a BST because the left child is less than its parent
  - This tree is a max heap because the parent is greater than all its children and the tree is complete



# Exercise

Create a valid BST containing the numbers 1-7 that:

- Has a depth of 2
- Has a depth of 3
- Has a depth of 4
- Has a depth of 5
- Has a depth of 6

What insertion order leads to the minimum depth? The maximum depth?