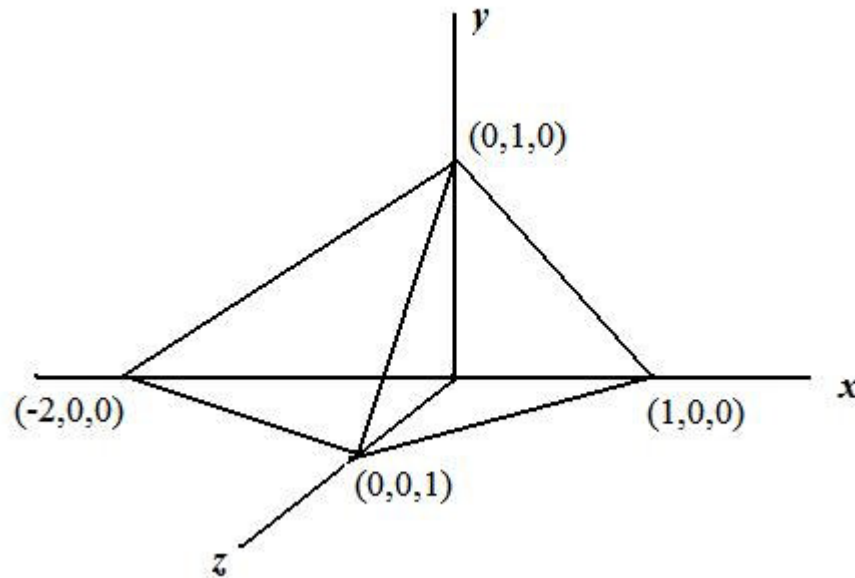


Instructions: Answer all three questions in the bluebook provided. 75 min, closed book/notes.

1.



(a) Construct a polyhedral volumetric representation for the given 4-sided figure. Specify all elements of the representation explicitly.

(b) Construct a sweep representation for the given tetrahedron. Specify all elements of the representation explicitly.

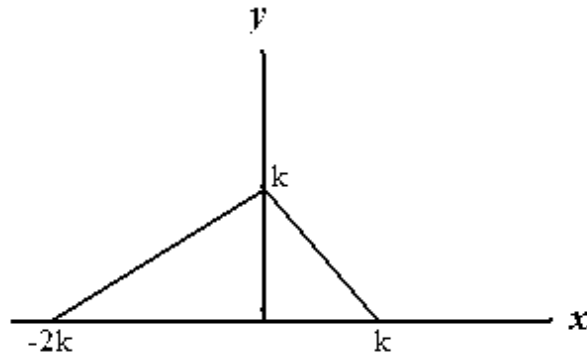
(a) The representation consists of the intersection of four half-space inequalities defined by their bounding planes. A plane is defined by the equation $ax+by+cz=d$. Two of the planes are $y=0$ and $z=0$. The third plane, the slanted one on the right, contains the three points $(1,0,0)$, $(0,1,0)$, $(0,0,1)$. This is $x + y + z = 1$ as can easily be seen by plugging each of the three points into that equation. Similarly, the fourth is $-\frac{1}{2}x + y + z = 1$ which can be checked the same way. So the four half-spaces whose intersection is the figure are:

1. $A_1: \{ (x,y,z): y \geq 0 \}$
2. $A_2: \{ (x,y,z): z \geq 0 \}$
3. $A_3: \{ (x,y,z): x+y+z \leq 1 \}$
4. $A_4: \{ (x,y,z): -1/2 x+y+z \leq 1 \}$

and the representation is $\bigcap_{i=1}^4 A_i$

(b) A sweep representation is defined by a spine $s(k)$ and a crosssection $c(k)$. each functions of a location parameter along the spine.

For the spine, let's pick the line segment $k(0,0,1)$ for $0 \leq k \leq 1$ (many others could be selected, this is just a convenient one since its crosssections are easily expressed). Then for any $0 \leq k \leq 1$, the crosssection at the level $z=k$ along the spine looks like



The two slanted edges are $y = -x + k$ and $y = \frac{1}{2}x + k$. So the sweep representation is defined by the following pair of functions of k :

Spine: $s(k)=k(0,0,1)$ for $0 \leq k \leq 1$;

Crosssection: $c(k)=\{ (x,y,z): y \geq 0, y \leq -x+(1-k), y \leq 1/2 x+(1-k) \}$

2. Suppose we use Nelson's associative memory based system to recognize two classes of objects: cubes and elongated cubes. Cubes are 3-D rectangles with length=width=height, elongated cubes have length>width, width=height. Assume some facets of each are covered in red wool, some in green wool, some in red paper, the rest in green paper.

(a) What would be a good selection of the set of keys if we expected considerable noise, clutter and occlusion when using the recognizer? Explain.

(b) Describe a single-pass associative memory algorithm that would use these same keys. How would the training of this system differ from training of Nelson's two-pass scheme?

(c) What is the advantage of the two-pass approach of Nelson's over a single-pass approach?

(a) Keys should be *local* and *semi-invariant*. One set of keys could be locations of the visible vertices, ie. intersections of edges. In addition, color and texture would be useful keys because these are also local and semi-invariant. Color and texture could particularly help out if many of the vertices were occluded. Unlike vertices, they are not only local and semi-invariant, they are also available at any visible location on a facet. Note that adding color and texture keys at each visible location would increase the dimension of the key feature vector a great deal.

(b) For each possible object type, pose, location and scale you could extract the keys and store as a vector in a reference database of vectors labelled by object type. For each test image you could find the reference image in the database that is closest in the associate memory sense, that determines your

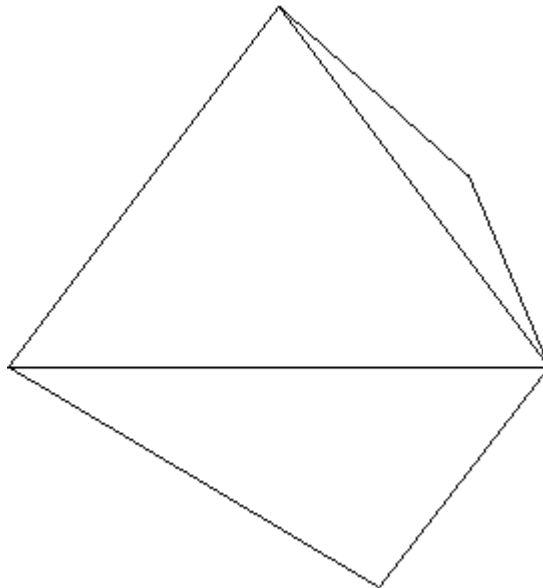
decision of object type. This training differs from Nelson's two-pass scheme in that probabilities and scales are not calculated.

(c) First pass of Nelson's two-pass scheme does not contain scale information or use probabilities, and since there is a second pass to follow, the threshold for success in this pass could be set low. Output of this pass is the set of objects and poses which are somewhat similar to the test images. Then scale (and in general other geometric) information is added back in to create the set of candidate hypotheses which form the evidentiary set. These are then input to the second pass, which considers all these factors, including probabilities. Most of the hypotheses have been eliminated in the first pass, that is the point of the two-pass approach.

3. Consider the 3D polyhedral object P where all its six faces are identical equilateral triangles. In the view of P shown below, three of P's six faces are visible.

(a) How many topologically distinct characteristic views does P have? Justify your answer.

(b) Define the *span* of an aspect graph as the greatest distance between any two nodes, ie. the maximum number of edges that must be traversed to get between nodes in the graph. Find the span of the aspect graph of P.



(a) There is one characteristic view with a single face, one with two faces, two with three faces, and one with four. For instance, if we number the top three faces 1, 2, 3, and the bottom three faces 4, 5, 6 with 4 beneath 1 and 5 beneath 2, then 1 is a characteristic view, 12 is a characteristic view with two faces visible, 123 and 124 are distinct characteristic views with 3 faces, and 1245 is the sole characteristic view with 4 faces. So there are a total of 5 characteristic views of P.

(b) With each edge traversed in the aspect graph we add or subtract one visible face. So the longest path would be from aspect A to aspect B, where all the facets are in A or B but not both. Aspects 123 and 456 are one such example: $123 \rightarrow 12 \rightarrow 1 \rightarrow 14 \rightarrow 4 \rightarrow 45 \rightarrow 456$. Another is aspects 14 and 2356, for which a path is $14 \rightarrow 124 \rightarrow 1234 \rightarrow 234 \rightarrow 2345 \rightarrow 235 \rightarrow 236$. All such paths are of length 6, this is the span of the aspect graph.