

An Algorithm to Follow Arbitrarily Curved Paths

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The Problem

- Given a curved path, find the appropriate movement so as to advance along the path's center
- Must be able to moderate one's speed along path so as to minimize acceleration endured by the body
- Should be robust to noise and missing data

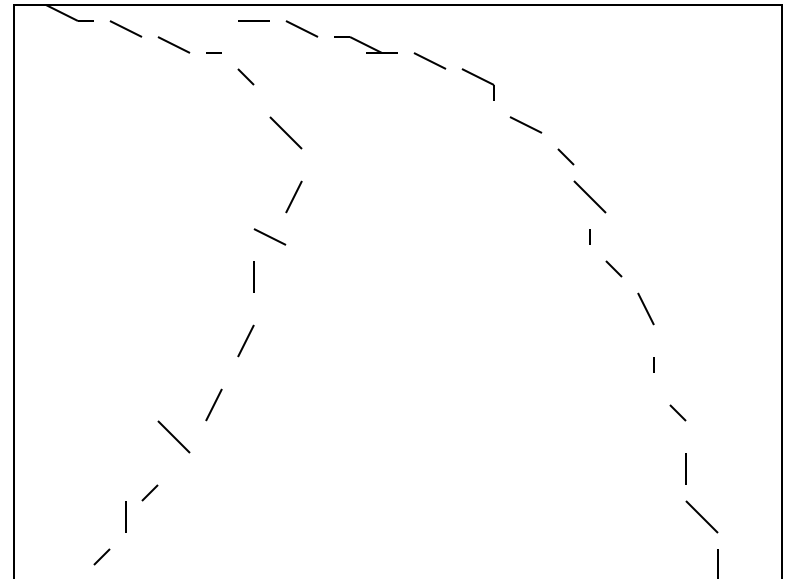
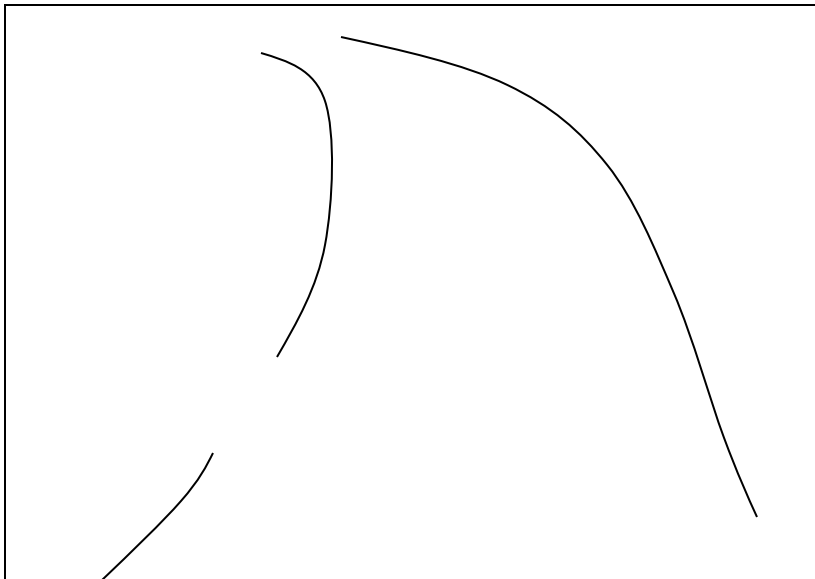


Solution Concept

- We assume image has been preprocessed to find image points of road edges
- Find a best fit curve for each road side
- Use slope of curves to estimate vanishing point
- Correlate radius of curvature to maximum safe velocity

Curve Fitting

- Use cubic spline curve to model road edge data
- Reduces noise compared to working with raw data
- Allows data interpolation when edge regions are missing





...Curve Fitting

$$\mathbf{c}(\lambda) = \mathbf{v}_1(1 - \lambda)^3 + \mathbf{v}_2 \lambda(1 - \lambda)^2 + \mathbf{v}_3 \lambda^2(1 - \lambda) + \mathbf{v}_4 \lambda^3$$

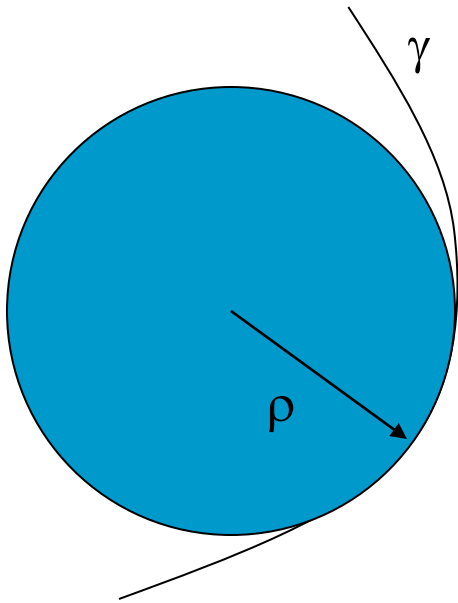
- Minimize objective function:

$$E(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4) = \sum_i |\mathbf{c}(\lambda_{min,I}) - p_i|^2$$

- Currently using mean squared error (may change to weighted mean squared error as appropriate)
- For fixed \mathbf{v} , $|\mathbf{c}(\lambda) - p_i|$ one can find $\lambda_{min,I}$ by numerically solving $D_\lambda |\mathbf{c}(\lambda_{min,I}) - p_i| = 0$ (Quintic Polynomial)
- Nonlinear optimization problem

Why look at curvature?

$$\rho = |\gamma'|^3 / \{ |\gamma'|^2 |\gamma''|^2 - (\gamma' \cdot \gamma'')^2 \}^{1/2}$$



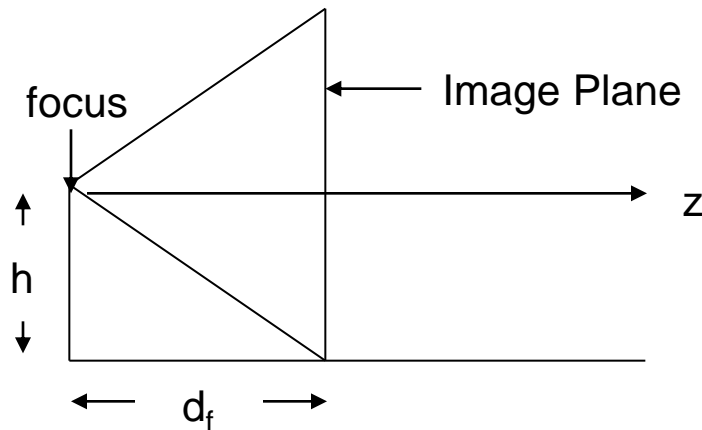
$$a_{\text{Transverse}} = v^2 / \rho$$

Need $a_{\text{Transverse}} < a_{\text{max}}$

$$a = a_{\text{Transverse}}^2 + a_{\text{parallel}}^2$$

...Curvature Continued

- Should be able to assume ground is level with optical axis for small z
- This allows us to find world curvature near the camera
- Not completely clear what to make of image curvature when z is not small





Literature Survey

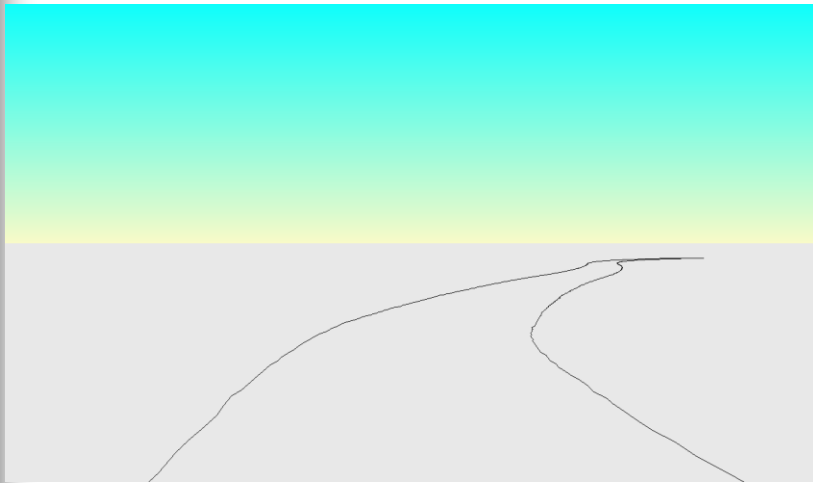
- Significant work done on edge detection: Ref. [R] focuses on straight roads but detects boundaries in highly non-ideal conditions
- Several papers explore curve fitting to determine vanishing points:
- Ref. [MDMT] explores a road model consisting of concentric circular arcs
- Ref. [WTS] Considers formation of cubic and quartic spline models
- Little work found on velocity control and curvature measurements



Approach Paradigm

- Task-oriented
- 3D *and* motion based vision
- Recovery only of select details

Completed Work



- Developed road simulation model
- Data structures to facilitate algorithms
- ~75% finished with curve fitting routine



Goals (certain to complete)

- Program vanishing point detection
- Continue investigating relationship between image curvature and world curvature
- Test algorithms with incomplete and noisy data



Goals (if time permits)

- Relax idealized parameters
- Test on real world data
- Add lane constraints
- Compare results with different road models
- Explore hybrid road models



References

- [WTS] Wang, Y., Teoh, E.K., & Shen, D. “Lane detection and tracking using B-Snake” *Image and Vision Computing* 22 (2004)
- [MDMT] Morgan, A.D., Dagless, E.L., Milford, D.J., Thomas, B.T. “Road Edge Tracking for Robot Road Following” *Image and Vision Computing* 8(3) (1990)
- [PS] Plass, M., Stone, M. “Curve-fitting with Piecewise Parametric Cubics” *Computer Graphics* 17(3) (1983)
- [R] Rasmussen, C. “Grouping Dominant Orientations for Ill-Structured Road Following” *Computer Vision and Pattern Recognition, Proceedings of the 2004 IEEE Computer Society Conference on*



...Curvature Continued

Possible Approach:

- Knowing curvature near camera gives maximum speed
- Can't change speed instantaneously:

$$a_{\text{Total}} = \{(\underline{v}^2 / \rho)^2 + (\Delta v / \Delta t)^2\}^{1/2}, \text{ where } \underline{v} = (v_1 + v_2) / 2, \Delta v = v_2 - v_1$$

Quartic equation for v_2 (has exact solution, though even this could be much simplified under certain assumptions)

- Correspond high curvature points between images
- Find time to collision with image plane