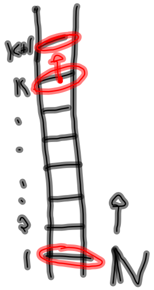

Induction

Thm: For any positive integer n,

$$1+2+3+\ldots+n = \frac{n(n+1)}{2}$$

Proof: Let $P(n)$ be the statement "$1+2+\ldots+n = \frac{n(n+1)}{2}$".

Base Case: $n=1$

$$P(1): \; 1 \overset{?}{=} \frac{1(1+1)}{2} = 1 \quad \checkmark$$

Induction Hypothesis: Assume $P(k)$ is true

$$\Rightarrow 1+2+\ldots+k = \boxed{\frac{k(k+1)}{2}}$$

Inductive Step: Show $P(k+1)$ is true

$$\boxed{1+2+\ldots+k}+(k+1) = \frac{k(k+1)}{2}+(k+1)$$

$$= \frac{k(k+1)}{2}+\frac{2(k+1)}{2} = \frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$\square$

Direct Proof

Logical Equivalent: $p \to q$

Thm: If a divides b, and b divides c, then a divides c.

p q r

$(p \wedge q) \to r$

Proof Idea: We will prove directly using the def. of divisibility.

Proof: By our assumptions that $a|b$ and $b|c$, there are natural numbers k_1 and k_2 such that:

Def. of Divis. $\Rightarrow$

$$\boxed{b} = \boxed{a \cdot k_1}$$
$$c = \boxed{b} \cdot k_2$$

$\Rightarrow c = (ak_1)k_2 = ak_1k_2$

Let $k = k_1 \cdot k_2$

$\Rightarrow c = a \cdot k$

And by the def. of divisibility, $a|c$. $\square$

Proof By Contradiction

Assume p (T) Trying to show $p \to q$ (T)
Assume $\neg q$ (T)
Try to find a contradiction such as $(r \wedge \neg r)$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Thm: $\sqrt{2}$ is not rational.

#'s representable as fractions

Proof Idea: By contradiction.
Assume that $\sqrt{2}$ is rational. and find a contradiction.

Proof: Assume $\sqrt{2}$ is rational.
So $\sqrt{2}$ can be represented as a fraction.
So $\sqrt{2} = \frac{a}{b}$ for some non-zero integers a & b such that a & b have no common divison. (Def. of Rational #)

Algebra
$\Rightarrow a = b\sqrt{2}$ square both sides
$\Rightarrow a^2 = b^2 \cdot 2$

Since 2 divides the RHS, then 2 also divides the LHS.
$\Rightarrow a^2$ is even.
$\Rightarrow a$ is also even.

So we can write a as $a = 2c$ for some $c \in \mathbb{Z}$

Substituting: $a^2 = 2b^2$
$$\Rightarrow (2c)^2 = 2b^2$$
$$\Rightarrow \cancel{4}^{2} c^2 = \cancel{2}b^2$$
$$\Rightarrow b^2 = 2c^2$$

So 2 divides b^2 by the same argument as before. So 2 divides b, so b is even.
So $\sqrt{2} = \frac{a}{b}$, but a and b both even.
So they share a common divison of 2. ✗