CSE 331: Introduction to Algorithm Analysis and Design Fall 2010

Homework 4

Due Friday, October 8, 2010 by 1:15pm in class

IMPORTANT: Please submit each problem separately, i.e. each problem should begin on a new page and only the pages for one problem should be stapled together. Failure to do so might result in some problem(s) not being graded.
For general homework policies and our suggestions, please see the policy document.

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1. (40 points) Let $G = (V, E)$ be an undirected graph. Let $u \neq v \in V$ be two vertices that are connected in G. Prove that there exists a *simple path* between u and v.

2. (45 points) Let $d \geq 1$ be an integer. Then a d-dimension *hypercube* is a graph whose vertex set is $\{0,1\}^d$. (Note that this implies that $n = 2^d$.) Further, a pair (u, v) is an edge if and only if the binary representations of u and v differ in exactly one of the d positions.

   Figure out a function $f(d)$ (for $d \geq 2$) such that the d-dimension hypercube has a cycle of length at least $f(d)$. (You will get more points the larger the value $f(d)$ is.) You have to prove the existence of a cycle of length at least $f(d)$ for any $d \geq 1$.

   (*Hint:* It might be helpful to assume the existence of the *Gray code*, which for any $\ell \geq 1$, outputs an ordering of binary vector of length ℓ such that one can go from one vector to the next one in the ordering by flipping exactly one bit (including from the last vector to the first).)

3. ($2 + 13 = 15$ points) Consider the following problem where the input are n numbers $a_, \dots, a_n$ and an integer $1 \leq k \leq n$. The goal is to output the k largest numbers in $a_1, \dots, a_n$. In this problem you need to do the following:

   - Design an algorithm to solve the above problem in time $O(nk)$.
   - Now design an algorithm to solve the above problem in time $O(n \log k)$.
     (*Hint:* Section 2.5 in the book might be helpful to solve the second part.)

   (*Note:* If you correctly solve the second part then you do not need to solve the first part.)