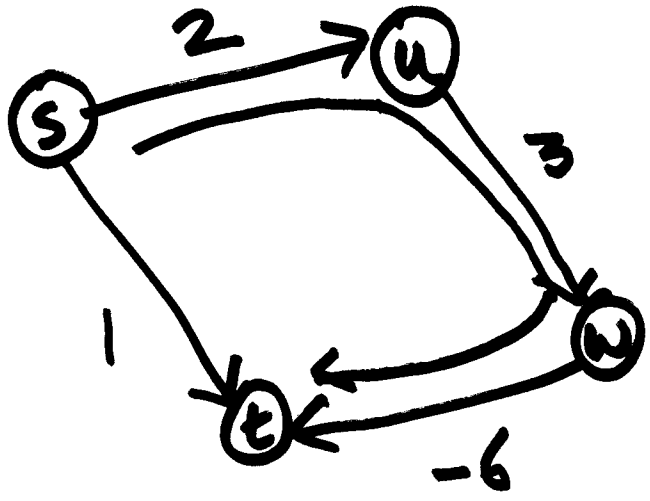
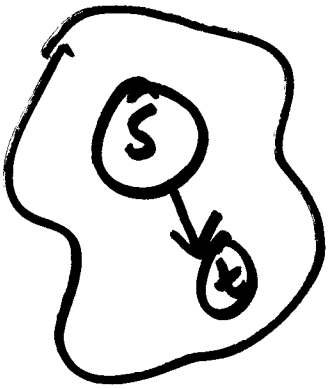
NOVEMBER 30, 2011



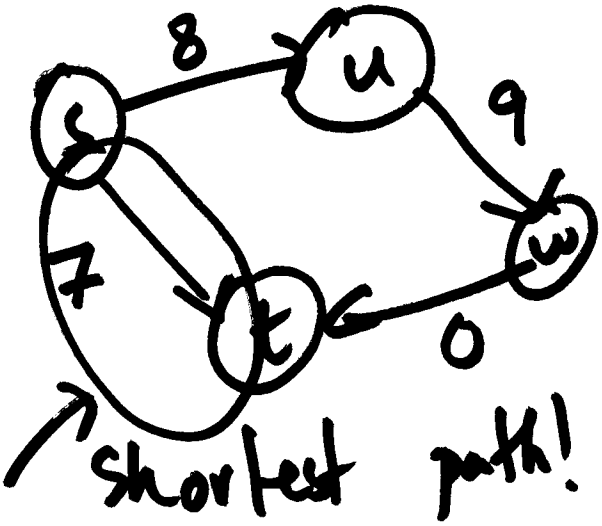
Shortest s-t path

Algo 1 (Dijkstra's)



t is picked 1st as $c_{s,t} < c_{s,u}$

Algo 2: Add +6 to all edge wts

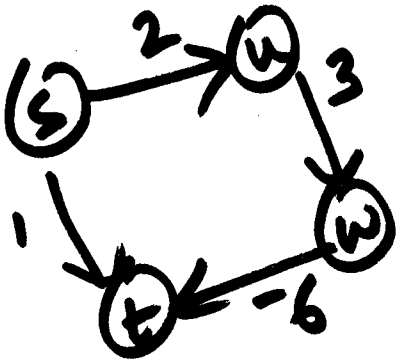


Shortest path changed!

Bad ✗

Group! Compute shortest s-t path with $\leq i$ edges

Ex. shortest st paths

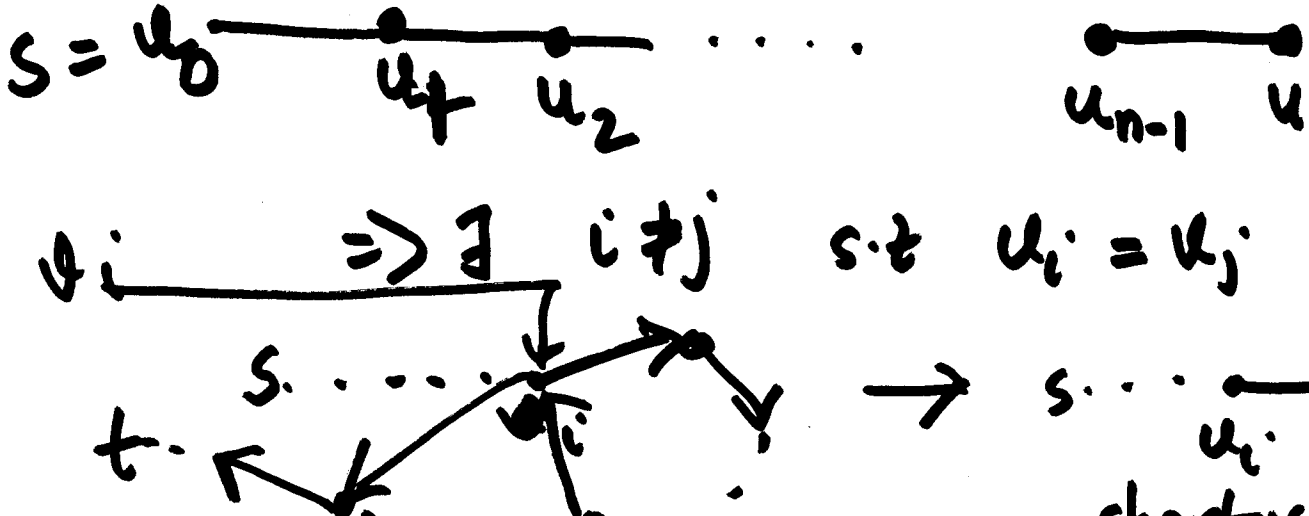


with
1 s-t : 1
2 " "
3 s-u-w : -1
4 ———
5 ———
6 ———
7 ———
8 ———

} No need to look at paths of length > 3

Prop: The shortest s-t path has $\leq n-1$ edges.

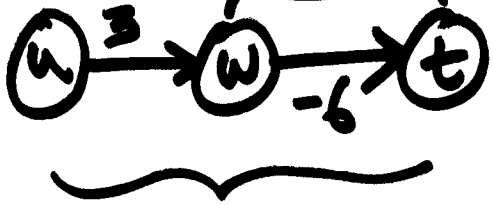
Pf: Ass. $\geq n$ edges



Dynamic Program

Idea: Keep track of all shortest u-t paths of length $\leq i$

v \ i	0	1	2	3
s	∞	1	1	-1
u	∞	∞	-3	-3
w	∞	-6	-6	-6
t	0	0	0	0



$OPT(i, u)$ → shortest u-t path $\leq i$ edges

$$OPT(i,u) = \min \left\{ OPT(i-1,u),\ \min_{(u,w) \in E} OPT(i-1,w) + c_{uw} \right\}$$

Base case:

$OPT(0,t) = 0$

$OPT(0,u) = \infty \quad \forall u \neq t$