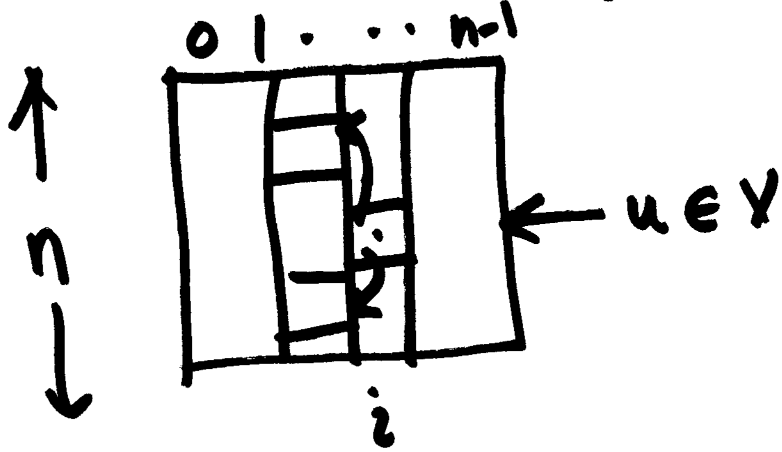
DECEMBER 2, 2011

$$OPT(i,u) = \min\left\{ OPT(i-1,u),\ \min_{(u,w)\in E} OPT(i-1,w) + c_{uw} \right\}$$



Column i <u>only</u> depends on column $i-1$

$M[u,i]$ $(= OPT(i,u))$ ← (prove by inducti

Bellman-Ford $(G = (V,E), \{c_e\}, t)$

1. $M[t,0] = 0$, $M[u,0] = \infty \ \forall u \neq t$ ← $O(n)$
2. For $i = 1 .. n-1$ ← n

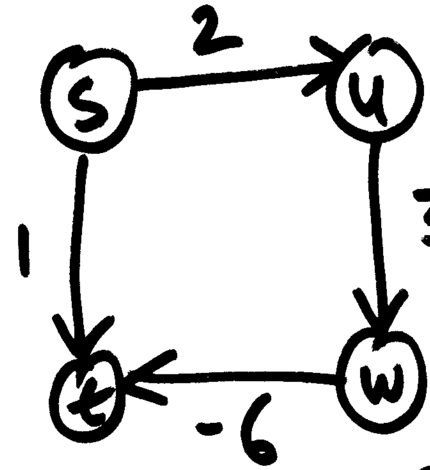
   For $u \in V$ ← n

   $$M[u,i] = \min\left\{ M[u,i-1],\ \min_{(u,w)\in E} c_{uw} + M[w,i-1] \right\}$$ $O(n_u)$ "$O(n)$"

$\boxed{M[s,n-1]}$

Q: How to figure out ~~len~~ cost of shortest s-t path (given M)

$\min(\infty, \min(1+0, 2+\infty)) = 1$



	0	1	2	3
s	∞	1	1	-1
u	∞	∞	-3	-3
w	∞	-6	-6	-6
t	0	0	0	0

shortest s-t path

$\min(\infty, \underline{3+(-6)})$

$\min(1, \min(1+0, \underline{2+(-3)}))$

$t \leftarrow w \leftarrow u \leftarrow s$

Run time:

$\rightarrow O(n^3)$

$$\rightarrow O\Big(n \cdot \underbrace{\sum_{u \in V} n_u}_{O(m)}\Big) = O(nm)$$

Ex: Algo to reconstruct shortest path

Q: Space: $O(n^2)$. Can you reduce this?

$O(n)$ space. Obs: i^{th} column only depends on $i-1^{th}$ column

Longest simple path problem

Read [KT, 6.8]