Dec 5, 2011

Longest Path $\in NP$

Input: $G=(V,E)$; $s \neq t \in V$

Output: 1 if $\exists$ a simple s-t path of length $n-1$
0 o/w

Witness: a path $P = v_1, v_2, \ldots, v_n$
Can verify if P is an s-t path of length $n-1$

Shortest Path $\in P$

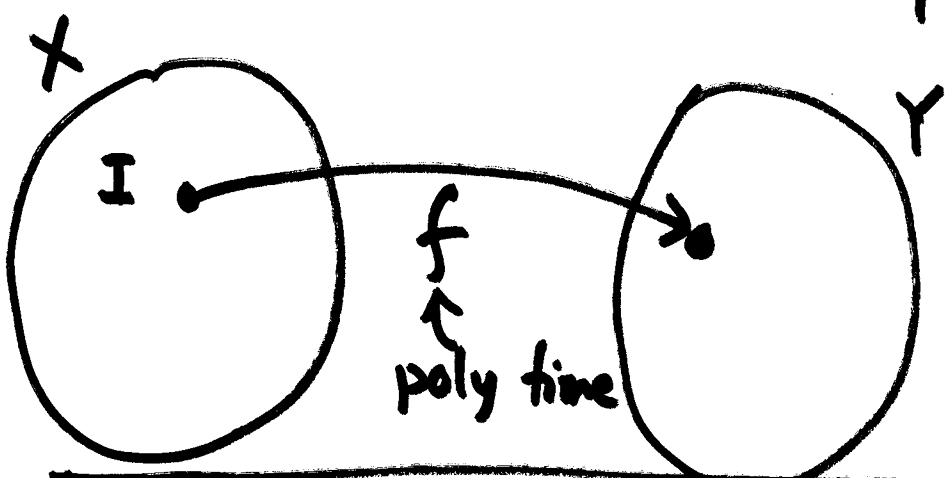
Input: $G=(V,E)$, c_e, $s \neq t \in V$, $C \in \mathbb{R}$

Output: 1 if $\exists$ an s-t path of cost $\le C$
0 o/w

REDUCTIONS

Problem X and Y

$X \le_p Y$ (X reduces to Y in poly time)



~~If~~ $\boxed{I \in X \iff f(I) \in Y}$

$\Rightarrow$ $Y \in P \Rightarrow X \in P$

E.g.: X → Hamburger joint

Y → shortest path problem

Showed: $X \le_p Y$

$\Rightarrow$ X is "hard" $\Rightarrow$ Y is "hard"

NPC

HAMILTONIAN - CYCLE (is NPC).

Input: $G = (V, E)$

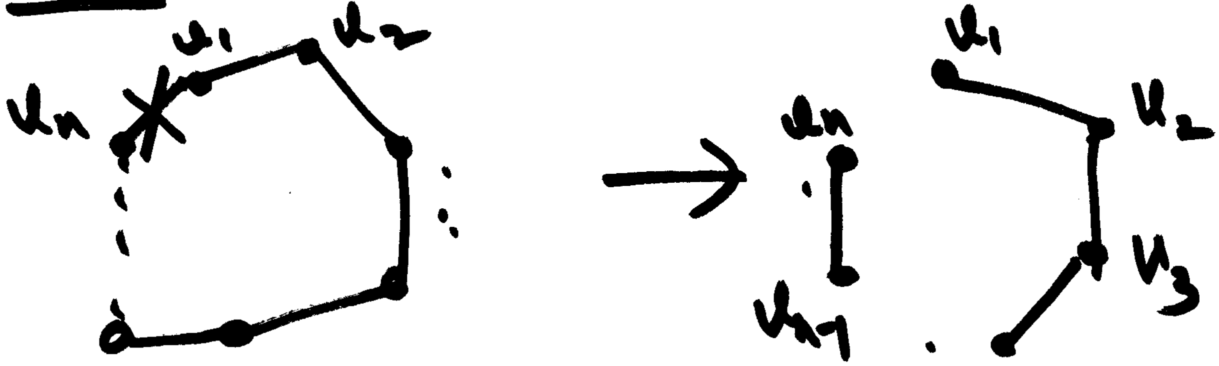
Output: 1 if $\exists$ a cycle through all n vertices

0 o/w

HAM-CYCLE $\leq_p$ LONGEST PATH

Overview:

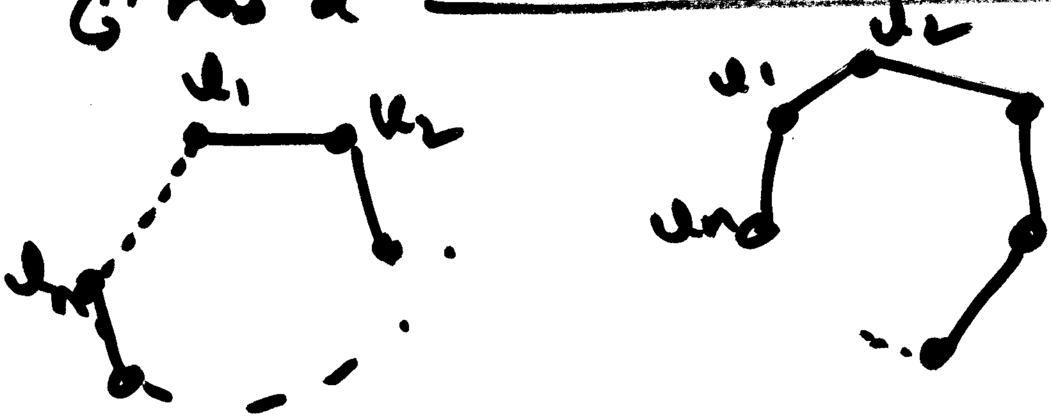
Case 1: G has a HAM-CYCLE



G': longest path of length $n-1$

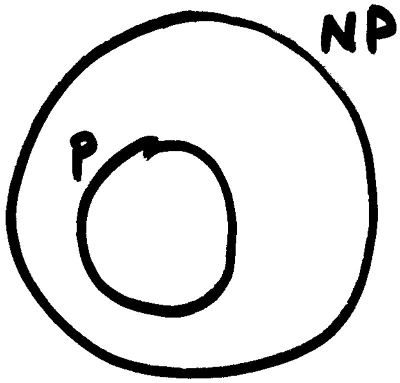
Case 2:

G' has a ______

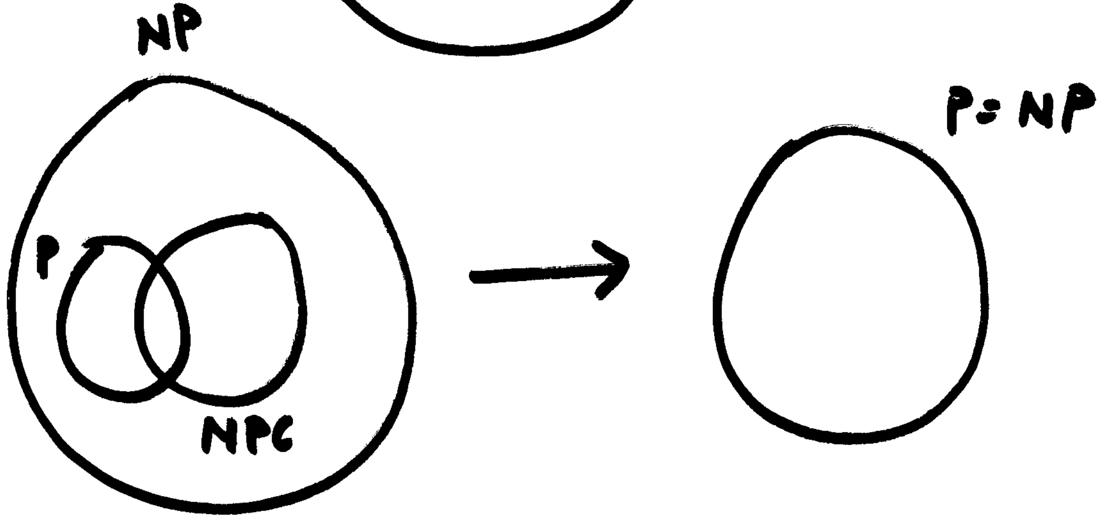


G has a HAM-CYCLE

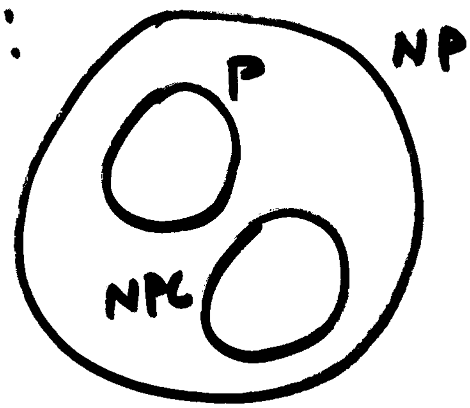
We know



Case 1:



Case 2:



→ Ladner's thm: If $P \neq NP \Rightarrow \exists\ L \in NP$ that is NOT NPC.

→ Candidates ($\in$ NP but probably not NPC)

→ Factoring

→ Graph Isomorphism