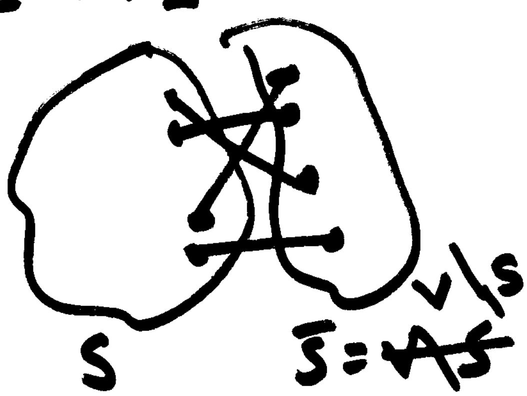
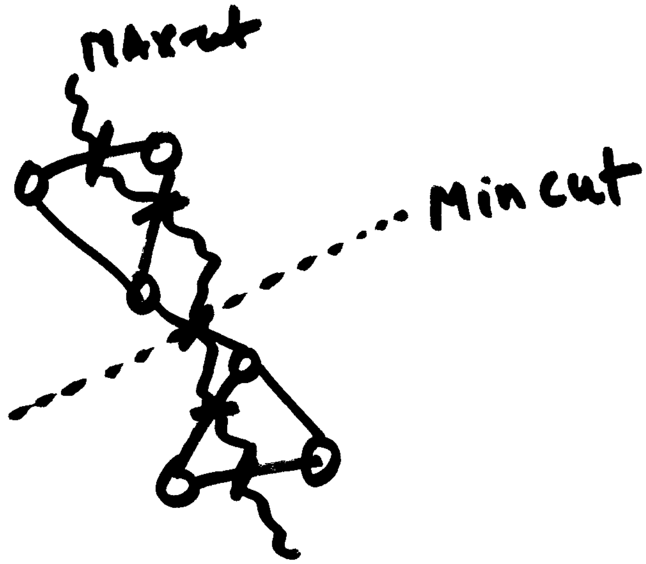
<u>DEC 7, 2011</u>

Q1: $|E(S,\bar{S})| = |E|$?

"Optimization problems"

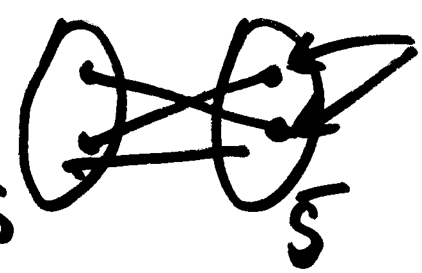
<u>MIN(IMUM) CUT</u>	<u>MAX(IMUM) CUT</u>
Input:	$G=(V,E)$
Output:	A "cut" $S \subset V$
<u>Goal:</u> min $|E(S,\bar{S})|$	max $|E(S,\bar{S})|$
$\in P$ "min cut max flow" [Ch. 7]	NP-complete.





$E(S,\bar{S})$ as the set of "crossing"/cut edges

<u>A1:</u> If $E(S,\bar{S}) = E$



Bipartite!

APPROXIMATION ALGOS

$\exists$ a poly time algo that outputs a cut of size

$$\left.\begin{array}{l} 99\% \\ 90\% \\ 50\% \\ 10\% \end{array}\right\} \text{~~as max~~ of } \underline{\text{OPT}}$$

Can do 87.856% [Goemans-Williamson 94]

IMPORTANT: Do $\boxed{\text{NOT}}$ know OPT.

$$\alpha = \min_{0 \le \theta \le \pi} \frac{2}{\pi} \cdot \frac{\theta}{1-\cos\theta} > .87856$$

hard to approximate with $\alpha+\varepsilon$ for ANY constant $\varepsilon > 0$

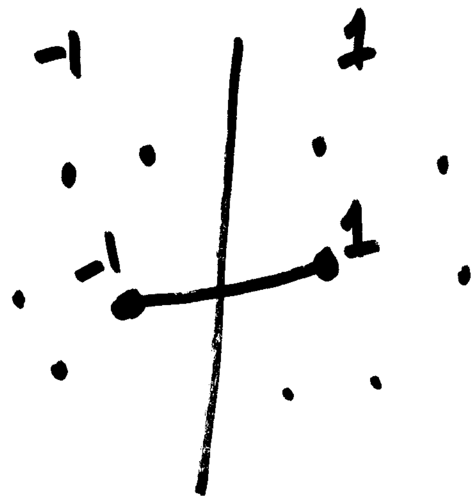
(via PCP Theorem + Unique Games Conjecture (UGC)) (circa 2005)

50% approximate algo

(RANDOMIZED ALGO)

CUT: $V \to \{-1, 1\}$

NOTE: Rid of random coins



size of cut = # $\underset{-1 \quad +1}{\bullet\!-\!\!-\!\bullet}$ or $\underset{+1 \quad -1}{\bullet\!-\!\!-\!\bullet}$

each $u \in V \to -1$ or 1

Randomized algo: Each $u \in V \leftarrow$ -1 with prob $\frac{1}{2}$, $+1$ with prob $\frac{1}{2}$

Claim: Expected size of the CUT $(G) \geq \frac{1}{2} \cdot OPT(G)$.

"Pf":

fix a edge $\underset{u}{\overset{-1}{\bullet}}\!-\!\!-\!\!\underset{w}{\overset{+1}{\bullet}}$ (or $u = +1$, $w = -1$)

$Pr[(u,w)$ is a crossing edge$] = \frac{2}{4} = \frac{1}{2}$.

$\Rightarrow E[\text{size of cut}] = \sum_{e \in E} Pr[e \text{ is crossing}] = \frac{|E|}{2} \geq OPT(G)$

But $|OPT(G)| \leq |E|$