CSE 331: Introduction to Algorithm Analysis and Design Fall 2012

HOMEWORK 6

Due Friday, November 2, 2012 by 1:15pm in class

IMPORTANT: Please submit each problem separately, i.e. each problem should begin on a new page and only the pages for one problem should be stapled together. Failure to do so might result in some problem(s) not being graded.

For general homework policies and our suggestions, please see the policy document.

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1. (40 points) Ms. SuperAgent is in a tough spot. She has two choices to neutralize the EvilestBadPersonEver. The first choice is to nuke EvilestBadPersonEver's hideout. The second choice is to accomplish n tasks that will neutralize EvilestBadPersonEver's security system (and then will make EvilestBadPersonEver capture a cakewalk). The ith task ($1 \leq i \leq n$) needs a given amount of time t_i and it has to be finished by time f_i. Each of these tasks are complex so Ms. SuperAgent can only do one task at a time and each task once it has been started has to be completed without interruption. EvilestBadPersonEver's security system is very smart so the only way to neutralize his security system is to finish *all* tasks i before time f_i.

   Ms. SuperAgent also has intelligence that EvilestBadPersonEver has planned an exit soon and she does not have time to try out both choices.

   Obviously Ms. SuperAgent wants to avoid the first choice because of the collateral damage. However, as mentioned earlier time is short and Ms. SuperAgent needs to decide if she can go with the second choice. During her training at the agency, she skipped the class on greedy algorithms. Your task, should you choose to accept it[1], is to design an algorithm for Ms. SuperAgent that will help her make her decision in time $O(n \log n)$.

2. (45 points) Ms. LiberalElite likes driving her Prius, which on one full tank of gas runs for 350 miles.[2] She is planning to drive from Buffalo to Seattle and has fixed up her driving route so that she can get to Seattle as fast as possible. She also has a map with the locations of all the n gas stations along the way. You can assume that the route is a straight line and the gas stations are points on the line. Ms. LiberalElite missed the class on greedy algorithms, so in this problem you will design an efficient algorithm for her, which she can use to figure out which gas stations she should stop at so that she stops at the minimum number of gas stations. Of course your algorithm should give a feasible set of gas stations, i.e. it should never be the case that is she is stranded between two gas stations without any gas. (You may assume that Ms. LiberalElite fills up the tank of her Prius whenever she stops at a gas station and that no two gas stations are more than 349 miles apart. Also you can assume that she starts off from Buffalo with a full tank.)

   Prove the correctness of your algorithm and analyze its running time. (For the latter, you will get more credit the smaller the running time of your algorithm.)

   Hint: Think of a greedy way to decide on which gas stations to stop. It might help to forget about the scheduling algorithms we have seen in class and just think of a greedy algorithm from "scratch." Then try to analyze the algorithm using similar arguments to one we have seen in class.

[1] In real life the choice of not accepting this mission of course is that you will lose the 40 points for the question.

[2] Lest you think I have anything against liberals or Prius, let me assure you that my brother-in-law has "termed" me and my wife as being liberal elites. And yes, we do have a Prius.

3. (15 points) Exercise 6 in Chapter 4.

   Note: As has been mentioned earlier, in many real life problems, not all parameters are equally important. Also sometimes it might make sense to “combine" two parameters into one. Keep these in mind when tackling this problem.

   Hint: In the solution that I have in mind, the analysis of the algorithm’s correctness follows the exchange argument.