NAME: ____________________

# CSE 331

## Introduction to Algorithm Analysis and Design

## Sample Mid-term Exam: Fall 2012

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DIRECTIONS:

- Closed Book, Closed Notes except for one $8\frac{1}{2}$" × 11" review sheet.
- Time Limit: 50 minutes.
- Answer the problems on the exam paper.
- Each problem starts on a new page.
- Make sure you write your NAME on the paper.
- If you need extra space use the back of a page.
- Problem 3b is a bonus problem.

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|---|---|
| 1a | /10 |
| 1b | /10 |
| 1c | /10 |
| 1d | /10 |
| 2 | /45 |
| 3a | /15 |
| 3b | /10 |
| Total | /100 |
| Bonus | /10 |

FEW GENTLE REMINDERS:

- You can quote any result that we covered in class or any problem that was there in a homework (but remember to explicitly state where you are quoting a result from).
- If you get stuck on some problem for a long time, move on to the next one.
- The ordering of the problems is somewhat related to their relative difficulty. However, the order might be different for you!
- You might be better off by first reading all questions and answering them in the order of what you think is the easiest to the hardest problem. Keep the points distribution in mind when deciding how much time to spend on each problem.
- Spend time on the bonus problem only if you are done with the rest of the exam.

1. ($4 \times 10 = 40$ points) Answer True or False to the following questions and briefly JUSTIFY each answer. A correct answer with no or totally incorrect justification will get you 4 out of the total 10 points. (Recall that a statement is true only if it is logically true in all cases while it is is false if it is not true in some case).

   (a) For any instance of the stable marriage problem with $n$ men and $n$ women, there are $n! = n \times (n-1) \times \ldots \times 1$ many possible perfect matchings.

   (b) Consider $f(n) = \log\log n$ and $g(n) = 10^{10^{10^{10^{10^{10}}}}}$. Then $f(n)$ is $O(g(n))$.

   (c) For any graph, there is a unique BFS tree for it.

   (d) Given a graph on $n$ vertices in its adjacency matrix, there is an $O(n^2)$ time algorithm to convert it into its adjacency list representations.

2. (20 + 25 = 45 points) Given an array $A$ of $n$ integers, consider the the following algorithm that computes a related value (and an intermediate matrix $B$):

```
For every $i = 1, \ldots, n$
    For every $j = i, \ldots, n$
        Assign $B[i, j]$ to be the maximum value among $A[i], A[i+1], \ldots, A[j]$.
Output the minimum value among all values in $B[i, j]$ (over all $i = 1, \ldots, n$ and $j = i, \ldots, n$).
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(a) Prove that the algorithm runs in $O(n^3)$ time.

(b) Present another algorithm that solves the same problem but runs in $O(n^2)$ time. (Briefly justify the running time and correctness of your algorithm.)

3. Let $d \geq 1$ be an integer. Then a $d$-dimension *hypercube* is a graph whose vertex set is $\{0,1\}^d$. (Note that this implies that $n = 2^d$.) Further, a pair $(u, v)$ is an edge if and only if the binary representations of $u$ and $v$ differ in exactly one of the $d$ positions.

   (a) (15 points) Figure out a function $f(d)$ such that the $d$-dimension hypercube has a cycle of length at least $f(d)$. (You will get more points the larger the value $f(d)$ is.) Briefly justify your answer.

   (*Hint:* You can assume the existence of the *Gray code*, which for any $\ell \geq 1$, outputs an ordering of binary vector of length $\ell$ such that one can go from one vector to the next one in the ordering by flipping exactly one bit.)

(b) (**Bonus**) (10 points) A *cut* of a graph $G = (V, E)$ is a partition of $V$ into two sets $S$ and $\bar{S} = V \setminus S$. The *value* of a cut $(S, \bar{S})$ (denoted by $E(S, \bar{S})$) is the total number of edges such that one end point is in $S$ and the other is in $\bar{S}$, i.e. it is the number of edges "crossing" the cut. The *maxcut value* of $G$ is

$$\max_{S \subseteq V} E(S, \bar{S}).$$

Figure out a function $g(d)$ such that the maxcut value of a $d$-dimension hypergraph is at least $g(d)$. Justify your answer. (To receive *any* credit for this problem, the function $g(d)$ has to depend non-trivially on $d$– at the very least it has to be asymptotically bigger than $d$. An answer without any justification will not receive any credit.)