

Final Exam Review

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Student asked Questions

Exchange Arguments

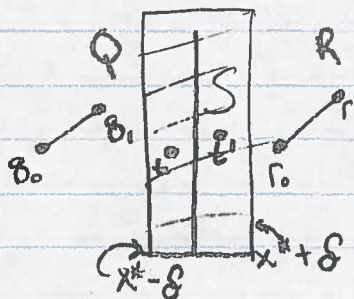
Ex: Prove the output of a greedy Algorithm is optimal
 g : Output of greedy σ : Output of Optimal
 $g_0 \rightarrow \dots \rightarrow g_m = \sigma$
(Series of Small Changes) $g_i \rightarrow g_{i+1}$

$\min \text{Cost}(\text{soln}) \quad \text{Cost } g_i \leq \text{Cost}(g_{i+1}) \quad \forall i \Rightarrow \text{Cost}(g_0 = g) \leq \text{Cost}(g_m = \sigma)$

ex) Lets say we want to maximize a profit.
 $\max \text{Profit}(\text{soln}) = \text{Profit}(g_i) \geq \text{Profit}(g_{i+1})$
 $\forall i \Rightarrow \text{Profit}(g) \geq \text{Profit}(\sigma)$
(=)

Kick Ass Lemma

- Divide Set of Points in half
- define delta (δ) =
 $\min(d(a_0, a_1), d(r_0, r_1))$

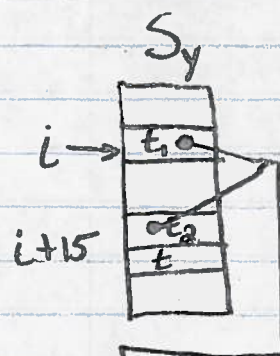


$\exists t \neq t' \in S$ such that
 $d(t, t') < \delta$

then t is at position i in S_y
 t' is at position j in S_y

$\Rightarrow |i - j| \leq 15$ homework (≤ 10)

Patchup: determine if such a pair (t, t') exists



Check if
 $d(t_1, t_2)$
is the Smallest!

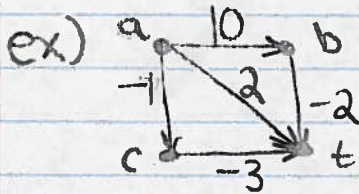
Shortest Path

given terminal t

$OPT(u, i) \rightarrow$ Cost of shortest $u-t$ path with $\leq i$ edges

$$OPT(u, i) = \min(OPT(u, i-1), \min_{(u,w) \in E} C_{u,w} + OPT(w, i-1))$$

$i > 0$



| | | | | |
|---|----------|---|---|---|
| | 0 | 1 | 2 | 3 |
| a | ∞ | 2 | | |
| b | ∞ | | | |
| c | ∞ | | | |
| t | 0 | | | |

↑
interested in this column

$$OPT(a, 1) = \min(\infty, \underset{\substack{\uparrow \\ b}}{10 + \infty}, \underset{\substack{\uparrow \\ t}}{2 + 0}, \underset{\substack{\uparrow \\ c}}{-1 + \infty}) = 2$$

Continue to apply formula to fill the chart.

• Note: you only need to know up to the $i-1$ th column.

Sample Final Problem 1 B **ERROR IN SOLUTION**
acyclic graphs!! ACTUAL ANSWER: FALSE!!!

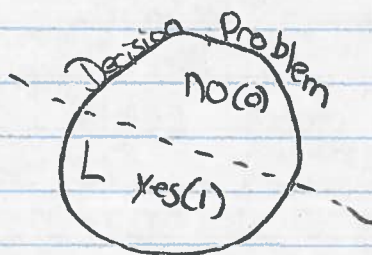
P = NP From Final Lecture "Decision Problems"

ex) not what is the longest path

P
(Polynomial Time)
• Problems that can be
Solved in
DTIME(n^c)

NP
(Non Deterministic Polynomial Time)
• Problems with polynomial time witness
 $L \in NP \iff \exists$ a poly time predicate R
Such that $x \in L$

question: is x a yes instance.



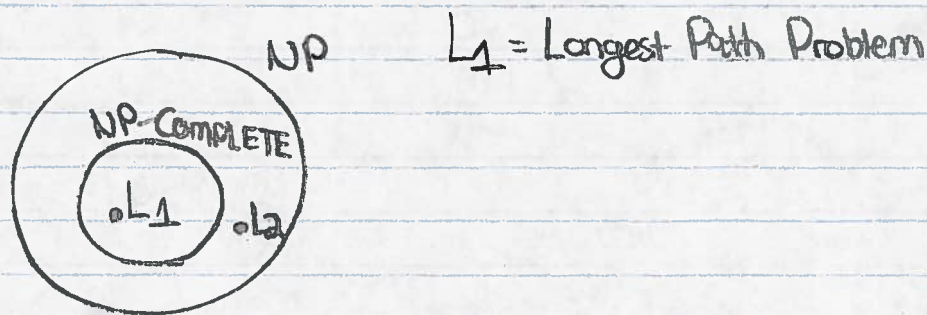
ex) $L_{LP} = \{G \mid \text{s.t. } \exists \text{ a simple path of length } n-1 \text{ in } G\}$ (3)

if $G \in L_{LP}$
 $w \rightarrow$ an actual path

$R(G, w) = 1 \iff w$ is of length $n-1$

Question: $P \stackrel{?}{=} NP$

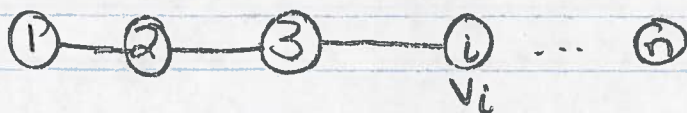
answer: WE DON'T KNOW!!



$L_1 \in P \Rightarrow \forall L \in NP, L \in P$

$\Rightarrow P = NP \leftarrow$ UNKNOWN IMPLICATION

Homework 10 Problem 1C



$OPT(j) =$ cost of max independent set in $1 - \dots - j$

$j \in \sigma_j \Rightarrow OPT(j) = v_j + OPT(j-2)$
 $j \in \sigma_i \Rightarrow OPT(j) = OPT(j-1)$

independent set
 no 2 points have an edge between them

Recursive Formula: $OPT(j) = \max(OPT(j-1), v_i + OPT(j-2))$

A tri's breaktime words of wisdom

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- go over the homeworks
 - there could be a variation of one of these problems.
- do try to solve a problem using another algorithm.
 - Change the input
 - Adjust accordingly
- decide what kind of algorithm to make
 - Greedy
 - Divide and cong.
 - Dynamic
- For problems with "don't have to solve a if you solve b"
 - Solve a anyways!! "You may not be 100% correct"
- Don't Panic!
 - if you get stuck for 15-20 mins, MOVE ON!
 - come back later!!!

