

Q1) Data structures for input

Q2) How do we find a free woman w ?

Q3) How does w pick her best unproposed man m ?

$O(1)$

Q4) How do we know who m is engaged to?

Q5) Does m prefer w to w' ?

Assumption: $M = \{1, \dots, n\}$ $m_i \rightarrow i$
 $W = \{1, \dots, n\}$ $w_i \rightarrow i$

Data Structures:

① Initialization time: (atmost $O(n^2)$)

② Query time (atmost $O(1)$)

③ Update time ($O(1)$)

A1) ManPref $[m, i] \rightarrow$ id of i th most preferred woman for m

WomanPref $[w, i]$

(Query time: $O(1)$
Update time: n/a)

Space: $O(n^2)$ man for w

A2) Linked list of all free women w .

Initialization: Add $\{1, \dots, n\} \leftarrow O(n)$

Query time: Pick first entry in the list
(& delete) $\leftarrow O(1)$

Update time: Add a free woman (first)
 $\leftarrow O(1)$

A3) Array Next with n entries

$\text{Next}[w] \rightarrow$ rank of the man she should propose to next.

Initialization: $\text{Next}[w] \leftarrow 1 \quad \forall w$
 $O(n)$

Query time: Read $\text{Woman Pref}[w, \text{Next}[w]]$
 $\leftarrow O(1)$

Update time: $\text{Next}[w]++ \leftarrow O(1)$

A4) Array Current

$\text{Current}[m] = w$ if m is engaged to ^{some} w

Initialization: $\text{Current}[m] \leftarrow -1 \quad \forall m$ (otherwise) $O(n)$

Query time: Read $\text{Current}[m] \quad O(1)$

Update time: Write to $\text{Current}[m] \quad O(1)$

Q 45) Scan m 's pref list ($O(n)$)

2-D array Rank n^2 entries

Rank $[m, w]$ $\rightarrow$ w 's rank in m 's pref list.

Query time: Compare Rank $[m, w]$ vs Rank $[m, w']$

Update time: n/a $O(1)$

Initialization: Scan n pref-list

each scan $O(n)$

$\Rightarrow O(n^2)$