PROPOSITION: Let T be any BFS tree for $G=(V,E)$

let $x \in L_i$, $\boxed{y \in L_j}$, $\boxed{(x,y) \in E}$

$\Rightarrow$ i & j differ by ≤ 1.

$(\equiv |i-j| \leq 1)$

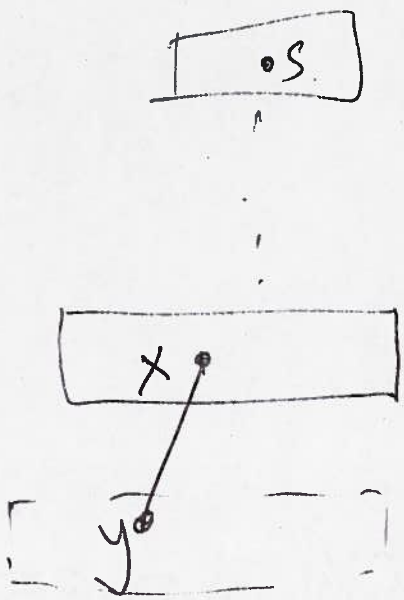
Pf: W.l.o.g. $i \leq j$ (otherwise switch x & y)

(Without loss of generality)

For contradiction assume $\boxed{i < j-1}$.

$\equiv$

$\boxed{j > i+1}$



L_0

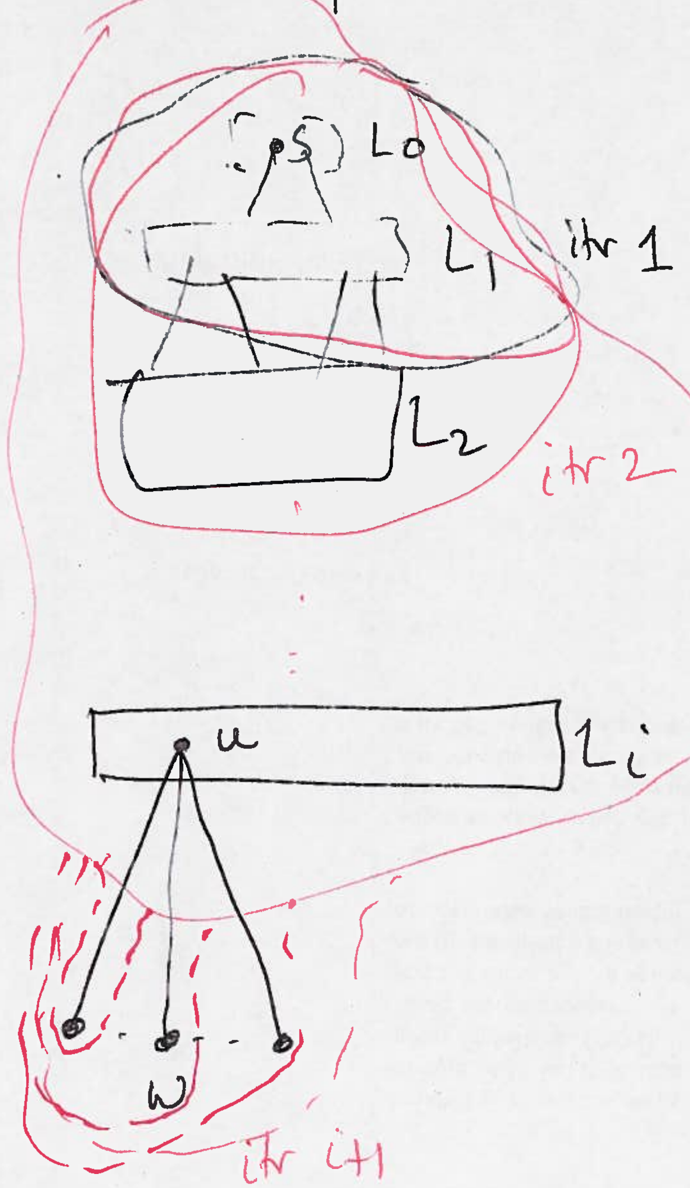
L_i $(y \notin L_0, \ldots, L_{i-1})$

By algo construction,

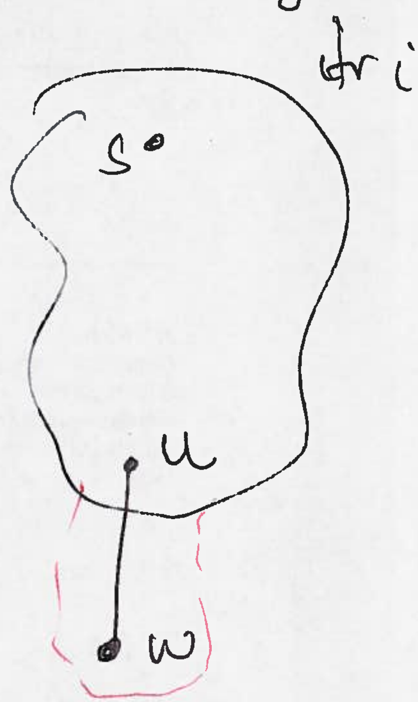
$y \in L_{i+1}$

Contradiction $\rightsquigarrow$ $j > i+1$.

"Growth" of BFS



Generic Algo



Explore(s)

1. $R \leftarrow \{s\}$
2. While $\exists\, u \in R$, $\exists\, w \notin R$ s.t $(u,w) \in E$

   $R \leftarrow R \cup \{w\}$
3. Return $R^* \leftarrow R$

<u>THM:</u> R^* = Connected Component of s. (CC(s))

($\Longrightarrow$ BFS(s) outputs ________)

<u>Pf idea:</u> $R^* \subseteq CC(s)$

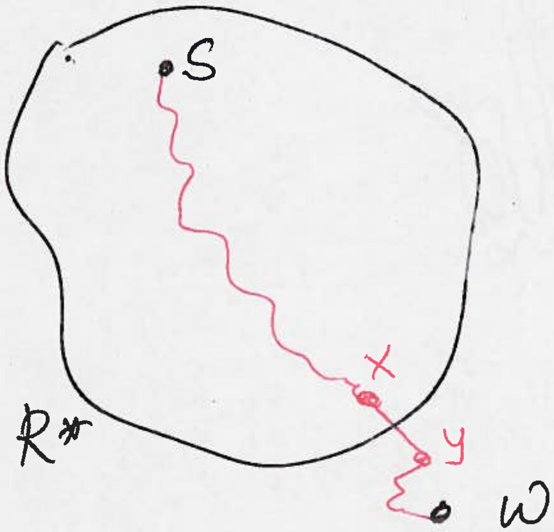
<u>Lemma 1:</u> $w \in R^* \Longrightarrow w \in CC(s)$ (Prove by induction on $|R|$)

$CC(s) \subseteq R^*$

<u>Lemma 2:</u> $w \notin R^* \Longrightarrow w \notin CC(s)$.

Pf of Lemma 2: For contradiction assume

$w \notin R^*$ but $s \rightsquigarrow w$



$s \in R^*$, $w \notin R^*$

$\exists$ a path fr s to w

$\Rightarrow \exists$ $\boxed{(x,y) \in E}$ in the path

$\boxed{x \in R^*}$

by $\boxed{y \notin R^*}$

y should be in R

$\Rightarrow$ the algo has not terminated.