$$\boxed{2m = \sum_{u \in V} n_u}$$

$G = (V, E)$

$|E| = m$

Adj list: n ptr + n linked lists

$$O(n) + \sum_{u \in V} O(n_u)$$

n_u: degree of u

$$= O\Big(n + \underbrace{\sum_{u \in V} n_u}_{2m}\Big)$$

$$= O(n+m) \text{ (adj list.)}$$

Adj matrix $O(n^2)$.

$m \leq \#\text{pair } (u,w)\ u \neq w$

$$= \frac{n(n-1)}{2} = \binom{n}{2}$$

$m \leq O(n^2)$

Adj. list is useful for <u>sparse</u> graphs

BFS (s)

O(n) 0) $CC[s] \leftarrow 1$, $CC[u] \leftarrow 0 \quad \forall u \neq s$

O(1) 1). $L_0 \leftarrow \{s\}$

2). $i \leftarrow 0$

3) $T \leftarrow \cancel{\phi}\ \{s\}$

O(n)

4) While $L[i] \neq \phi$ $\quad L[i+1] \leftarrow \phi$ ← $l_1 \leq n$] $\leq n$

O($l_1 l_2 l_3$) For* every $u \in L[i]$ ← $l_2 \leq n$

$\leq m$

For each edge $(u,w) \in E$ ← $l_3 \leq n_u \leq n$

O(1) If $CC[w] = 0$

$CC[w] \leftarrow 1$

Add (u,w) to T

Add w to $L[i+1]$

i++

Lemma: BFS above is $O(n^2)$ [$O(n^3)$]

While + For* loop runs atmost once per vertex $u \in V$

$\Rightarrow O(n^2)$ run time

Step 4 $\rightarrow O(\sum_u n_u) = O(m)$.

Overall $\rightarrow O(m+n)$