Dijkstra's algo

Array with current d[u] values.

0. $R \leftarrow \{s\}$, $d(s) \leftarrow 0$ $\leftarrow O(1)$

1. While $\exists x \notin R$ s.t. $(u,x) \in E$, $u \in R$

$\leq n$ iterations

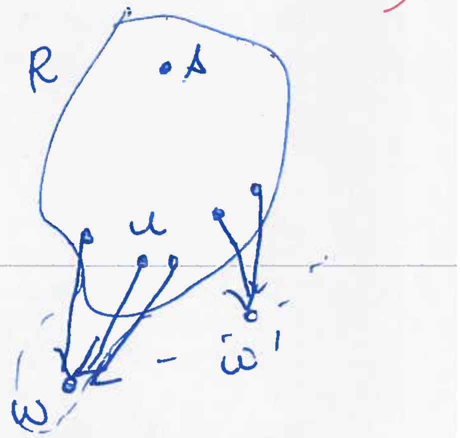
Pick w to be the min such x $\leftarrow \leq \sum_{w \in V} O(n_w^{in}) = O(m)$

by $d'(\cdot)$ value

$O(n_w^{in})$ $\quad O(1)$ $\begin{cases} \text{Add } w \text{ to } R \\ d(w) \leftarrow d'(w) \end{cases}$ $\quad \boxed{O(mn)}$

$$\boxed{d'(w) = \min_{\substack{u \in R \\ (u,w) \in E}} d(u) + l(u,w)}$$

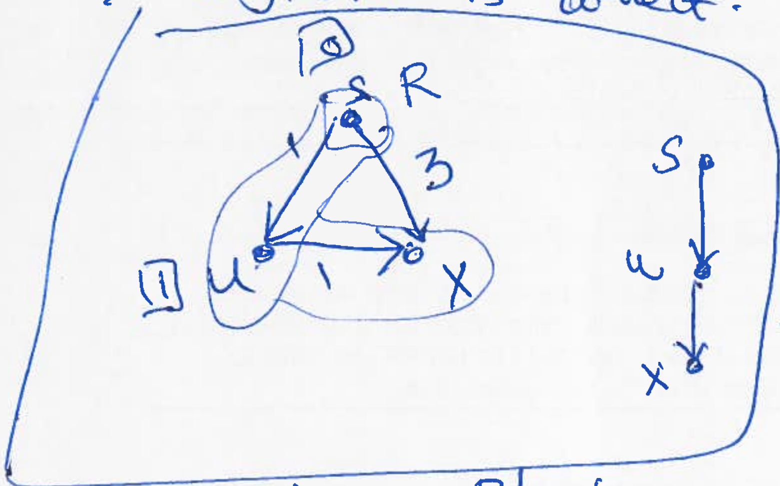
$O(n_w^{in})$



$P_u \rightarrow$ be the path from s to u in the "shortest path" graph.

<u>THM:</u> Let R be the set at any pt. in the run of the ~~alg~~ algo. Then P_u is a shortest s-u path for any $u \in R$.

$\Rightarrow$ Dijkstra's is correct. (Pick R at the termination of the algo)

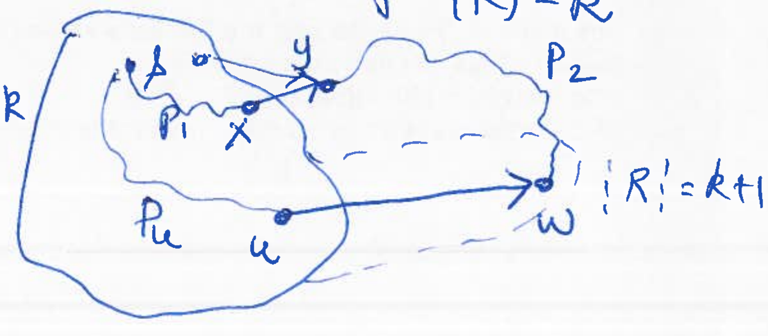


Induction on $|R|$.

<u>Base Ca:</u> $|R| = 1 \Rightarrow P_s$ is shortest ✓

Assume true for $|R| = k$

IS $\rightarrow$ Now $|R| = k+1$



Consider $P_w^* = P_u, (u,w)$

For contradiction assume $\exists\ P' \neq P_w$ s.t.

$\ell(P') < \ell(P_w)$

$P' = \underbrace{P_1}_{P_x}, (x,y), P_2$

Consider d' values before w is added

$d'(y) \leq d(x) + \ell_{(x,y)}$

$\leq \ell(P')$

$d(P_w) = d'(w) \leq d'(y) \leq \ell(P')$

as Algo picked w over y

$O(m \log n)$ implementation

Heap → Extract-min ($O(\log n)$)

Heap ↳ Change-key ($O(\log n)$)

Idea: Maintain heap on all $w \in V \setminus R$

value: $d'(w)$

Initialization: $d'(s) = 0$ $\quad d'(w) = \infty$ $\quad w \neq s$

Find min w: Extract-min ($O(\log n)$)

Update: Change-key operation.