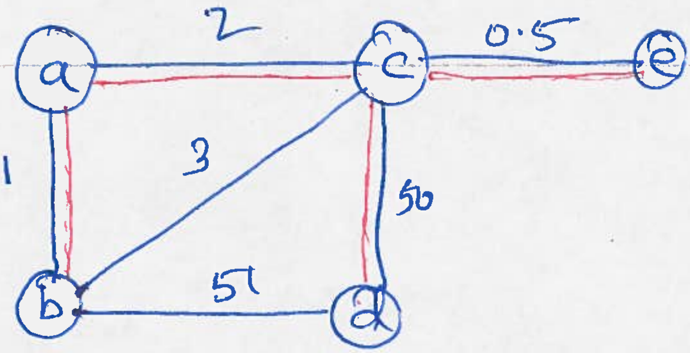
Minimum Spanning Tree (MST)

I/p: $G = (V, E)$ $\quad c_e > 0 \quad \forall e \in E$

O/p: $T \subseteq E$ s.t $G' = (V, T)$ is connected.

Goal: Minimize $\text{Cost}(T) \overset{\text{def}}{=} \sum_{e \in T} c_e$.

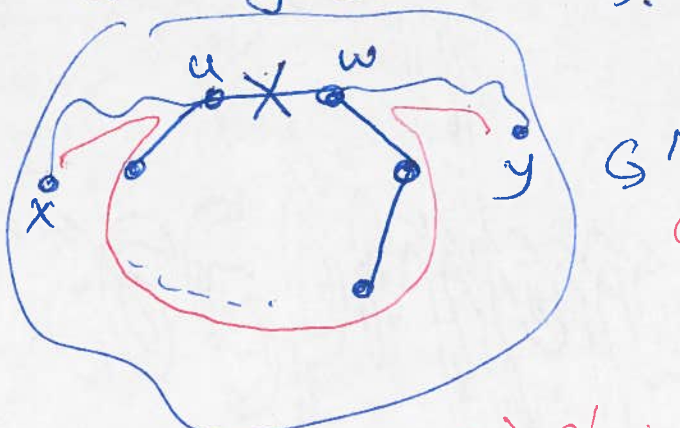


Prop: If G' has min cost, then G' is a tree.

Pf: By contradiction. Assum G' is not a tree.

$\Rightarrow \exists$ a cycle in G'!

$H \leftarrow G' \setminus \{(u, w)\}$



G'

H is connected.

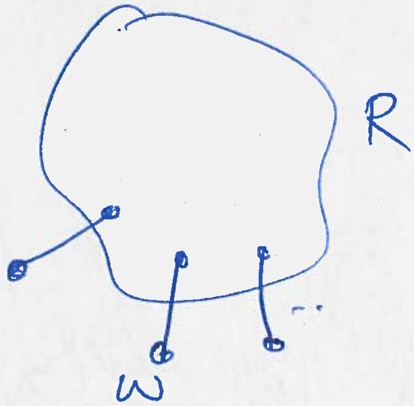
$\text{cost}(H) = \text{cost}(G') - c_{(u,w)} < \text{cost}(G')$

$\Rightarrow G'$ is not optimal $\times$

Prim's algo:

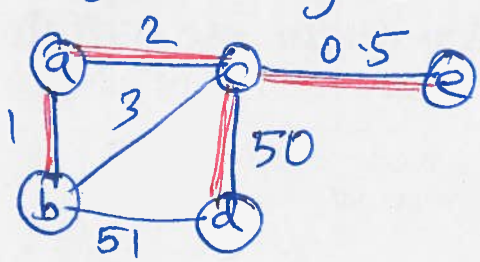
⤳ Build R iteratively

Pick w w/ min cost crossing edge



Kruskal's algo:

0. Sort edges by cost (incr. order)
1. Add them one by one (in order) unless you get a cycle



Reverse-Delete algo:

0. Sort edges by cost in decr. order
1. Drop edges one by one unless it disconnects the graph.

Correctness of Kruskal + Prim:

Assume: All edge costs are distinct.

$\boxed{\text{CUT PROPERTY LEMMA}}$:

$\forall \emptyset \neq S \subset V$

Cheapest crossing edge is in ALL MSTs of G.