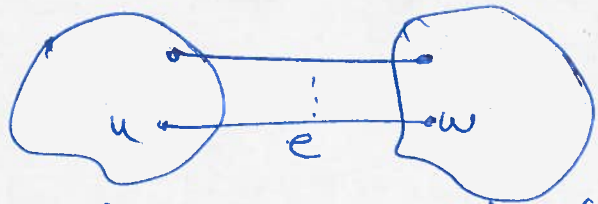
Input graph $G = (V, E)$ is connected.

CUT PROPERTY LEMMA: Assume all edge costs are distinct. $\forall$ ~~$S \subseteq V$~~ $\emptyset \neq S \subset V$

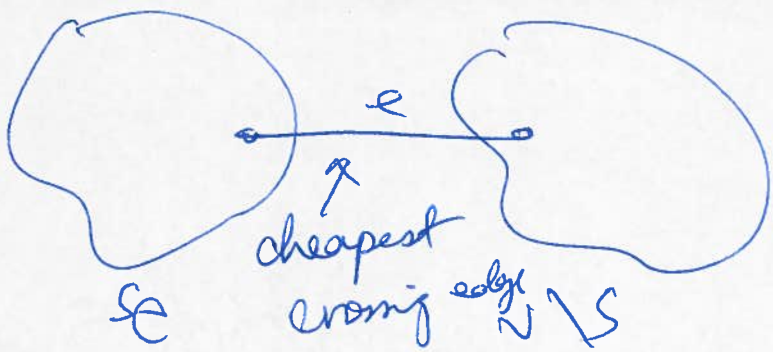


$\emptyset \neq S$ $\quad$ $V \setminus S \neq \emptyset$

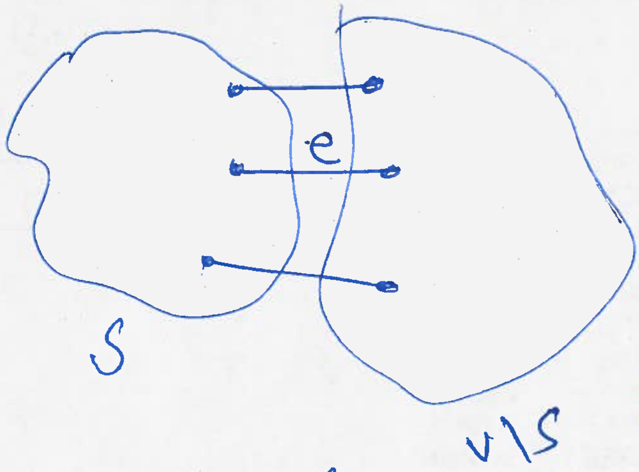
The cheapest crossing edge $e = (u, w)$ is in ALL MSTs of G.

Assume Cut Property Lemma is true.

High level idea: For every edge e that is added by K/P's algo, argue $\exists$ a cut S_e



Correctness of ~~Kruskal's~~ Prims algo



By defn Prim's algo picks the cheapest crossing edge.

Argn (1) The final output is a tree ← Ex. (induction on $|S|$)

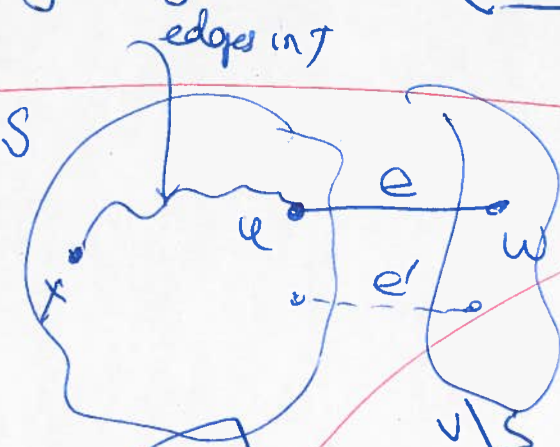
(2) $S \neq \phi$ ← (as $S \leftarrow \{s\}$ in step 0)

(3) $S \subset V$ ← (as long as algo has not terminated.)

Correctness of Kruskal's algo

Let's say K's algo adds edge $e = (u, w)$ to T.

S → set of all vertices connected to u by edges in T (<u>before</u> e is added)



(i) $S \neq \phi$ (as $u \in S$)

(ii) $S \subset V$ (w $\notin$ S)

(iii) e is the cheapest crossing edge

(iv) Final o/p (V, T) is connected.

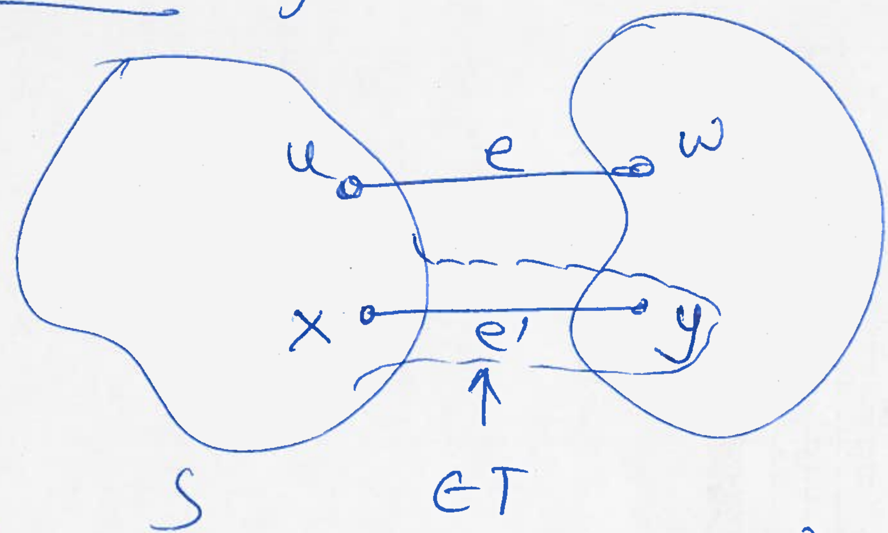
contradiction as adding e cannot create a cycle!

$\boxed{\text{(iii)}}$

General idea: edges are added in incr. order of cost.

Claim: e is the 1st crossing edge for (S, V\S) that the algo considers.

Pf of Claim' Say not

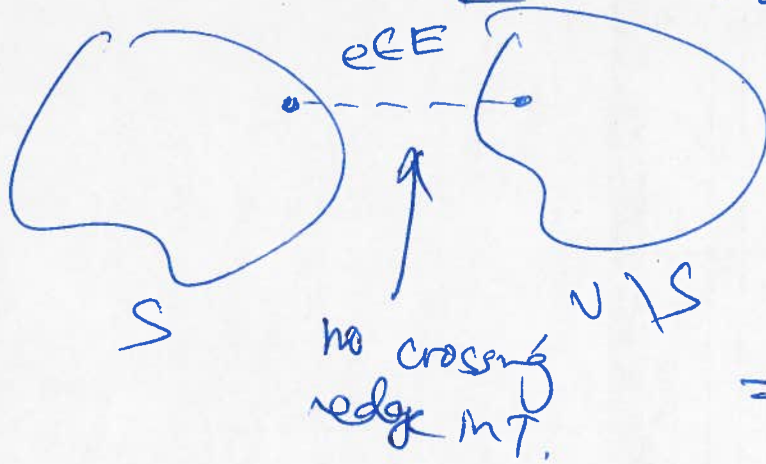


If a crossing edge $e' = (x, y)$ was already connected $\Rightarrow$ $y \in S \Rightarrow e'$ is not a crossing edge

(iv) Final output is connected.

By contradiction. (N, T) is 7 connected. as G is connected.

$\Rightarrow$



$\Rightarrow$ e could be added by the algo

$\Rightarrow$ ~~Alg~~ Algo has not terminated