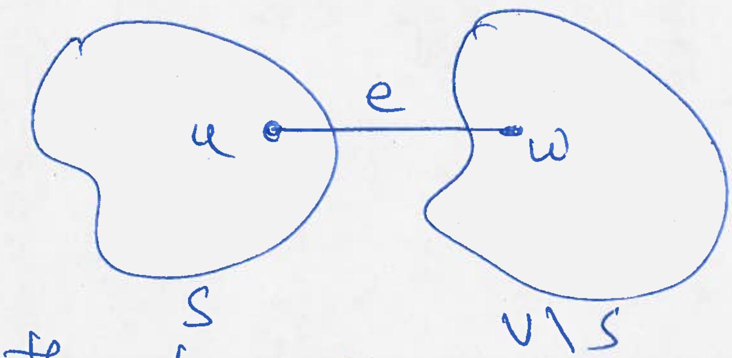
Cut Property Lemma $\quad G=(V,E)$

Assume all edge costs $c_e > 0$ are distinct.
Then <u>FOR EVERY</u> $\emptyset \neq S \subset V$

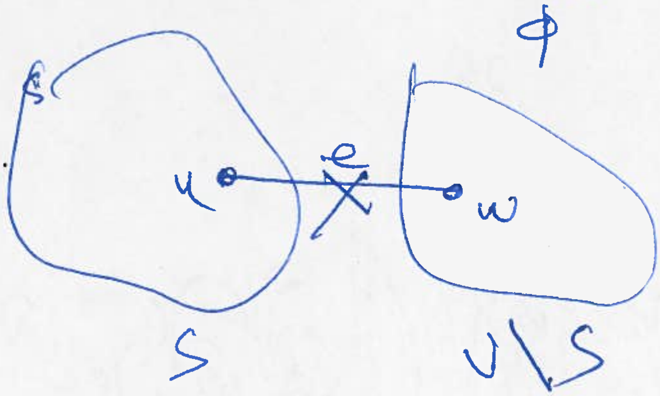


If e is the cheapest crossing edge, then e is in ALL MSTs for G

Pf (idea): By contradiction

Assume $\exists$ an MST (V, T')

& $\exists$ a cut $\underset{\neq \emptyset}{S}, \underset{\neq \emptyset}{V \setminus S}$



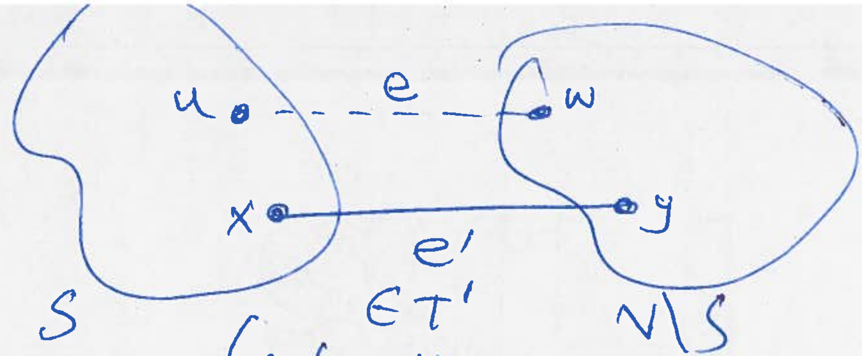
$e \notin T'$

$\Downarrow$ Exchange argument

come up with a T^*
s.t. (V, T^*) is a spanning tree
but $\mathrm{cost}(T^*) < \mathrm{cost}(T')$.

Exchange argument:

$\exists$ a crossing edge



$c_e < c_{e'}$

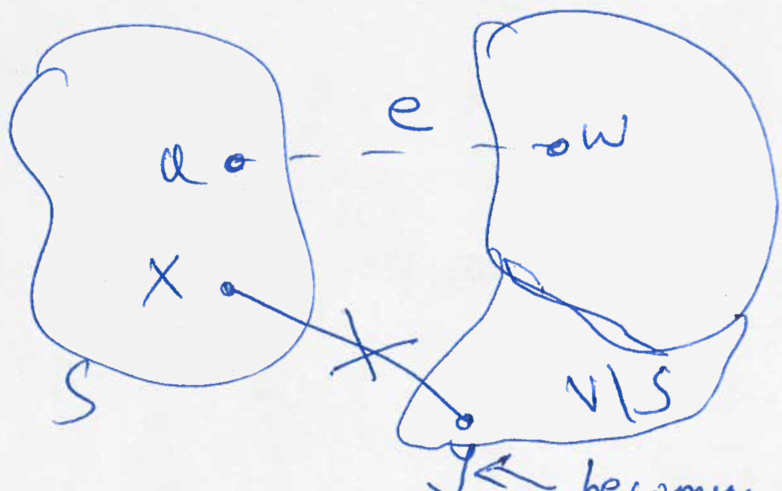
(as (V,T') is connected.)

Define $T^* = \left(T' \setminus \{e'\}\right) \cup \{e\}$

$$\begin{aligned} cost(T^*) &= cost(T) - c_{e'} + c_e \\ &< cost(T) \end{aligned}$$

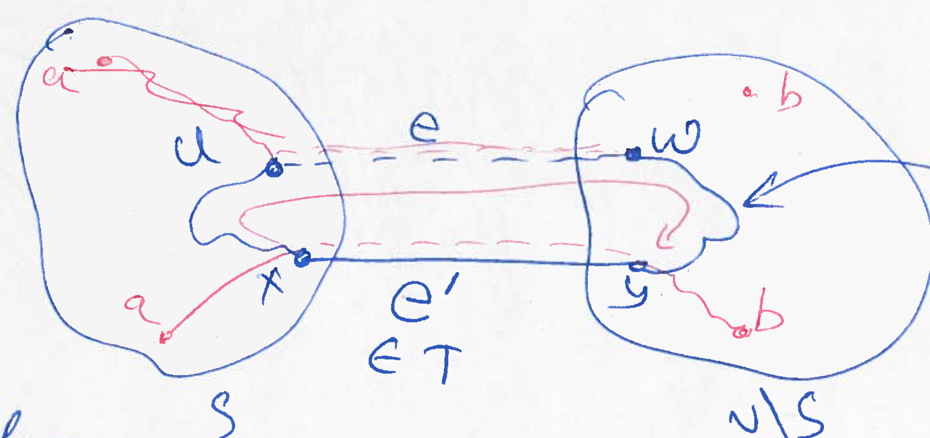
$\dashrightarrow$ (V, T^*) is not an MST

Bad case



y ← becomes disconnected!

Fix!



only edges in T'

T^* as before.

Perturbing edge costs.