

$$T(n) \leq c \quad n \leq 2$$

$$\leq cn + 2T\left(\frac{n}{2}\right)$$

$$\dots T(n) \leq O(?)$$

Lemma: $T(n) \leq \text{~~code~~} c \cdot n \cdot \log_2 n$

$\Rightarrow$ Mergesort runs in time $O(n \log n)$

$\rightarrow$ No known general sorting algo w/ run time asymp. better than $n \log n$

$\rightarrow$ Any sorting algo needs $\Omega(n \log n)$ comparisons.

$\rightarrow$ If each $a_i \leq O(n) \Rightarrow \nexists O(n)$ sorting algo (Ex.)

$\rightarrow$ Quicksort

Solving the recurrences

Strategy 1:

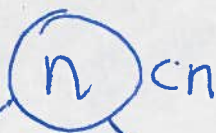
- (i) Unroll/unravel the recurrence for a few steps.
- (ii) Spot a pattern
- (iii) Generalize

Strategy 2: Guess & verify!
 $\uparrow$ (induction)

$$T(n) \leq c \quad n \leq 2$$

$$\leq cn + 2T\left(\frac{n}{2}\right)$$

level 0



cn

level 1

$\frac{cn}{2}$



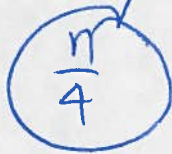
$\frac{cn}{2}$



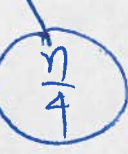
$$\frac{cn}{2} + \frac{cn}{2} = cn$$

level 2

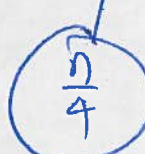
$\frac{cn}{4}$



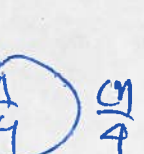
$\frac{cn}{4}$



$\frac{cn}{4}$



$\frac{cn}{4}$



$$4 \cdot \frac{cn}{4} = cn$$

level i

$\frac{cn}{2^i}$



$$2^i \cdot \frac{cn}{2^i} = cn$$

$$T(n) \leq cn (\underbrace{\# \text{levels}}_{\leq \log_2 n}) \leq cn \log_2 n$$

Strategy 2: Guess $T(n) \leq cn \log_2 n$

Base case: ($n=2$)

$$T(n) \leq c \cdot 2 \cdot 1 = 2c \quad \checkmark$$

Assume $T\left(\frac{n}{2}\right) \leq \frac{cn}{2} \log_2 \frac{n}{2}$

$$\begin{aligned} \text{Now } T(n) &\leq cn + 2T\left(\frac{n}{2}\right) \leq cn + 2 \cdot \frac{cn}{2} (\log_2 n - 1) \\ &= cn + cn(\log_2 n - 1) = cn \log_2 n \end{aligned}$$

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 $O(n^2)$ in general~~1~~010

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$$a = (a_{n-1}, \dots, a_0)$$

$$\text{dec}(a) = \sum_{i=0}^{n-1} a_i 2^i$$

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$$a^0 = a_{\lfloor \frac{n}{2} \rfloor + 1}, \dots, a_0$$

$$a' = a_{n-1}, \dots, a_{\lfloor \frac{n}{2} \rfloor}$$

$$\text{dec}(a) = \text{dec}(a') 2^{\lfloor \frac{n}{2} \rfloor} + \text{dec}(a^0)$$

$$\text{dec}(a \cdot b) = (a' 2^{\lfloor \frac{n}{2} \rfloor} + a^0) (b' 2^{\lfloor \frac{n}{2} \rfloor} + b^0)$$

$$= \underbrace{a'b'} 2^{2\lfloor \frac{n}{2} \rfloor} + (\underbrace{a'b^0} + \underbrace{a^0 b'}) 2^{\lfloor \frac{n}{2} \rfloor} + \underbrace{a^0 b_0}$$