$$ab = (a^1 2^{\lfloor \frac{n}{2} \rfloor} + a^0)(b^1 2^{\lfloor \frac{n}{2} \rfloor} + b^0)$$ } Dec(

$$= a^1 b^1 2^{\lfloor \frac{n}{2} \rfloor} \cdot 2^{\lfloor \frac{n}{2} \rfloor} + a^1 b^0 2^{\lfloor \frac{n}{2} \rfloor} + a^0 b^1 2^{\lfloor \frac{n}{2} \rfloor} + a^0 b^0$$

$$= a^1 b^1 2^{2\lfloor \frac{n}{2} \rfloor} + (a^1 b^0 + a^0 b^1) 2^{\lfloor \frac{n}{2} \rfloor} + a^0 b^0$$

$Dec(a) = a^1 2^{\lfloor \frac{n}{2} \rfloor} + a^0$ $\quad a = a_{n-1}, \ldots, a_0$

$Dec(b) = b^1 2^{\lfloor \frac{n}{2} \rfloor} + b^0$ $\quad a^0 = (a_{\lfloor \frac{n}{2} \rfloor - 1}, \ldots, a_0)$

$(c+d)(e+f) = ce+cf+de+df$ $\quad a^1 = (a_{n-1}, \ldots, a_{\lfloor \frac{n}{2} \rfloor})$

$a = 1101$ $\quad Dec(a) = 13$

$a^1 = 11$ $\quad Dec(a^1) = 3$

$a^0 = 01$ $\quad Dec(a^0) = 1$

$\frac{2^x}{2^y} = 2^{x-y}$

$2^x \cdot 2^y = 2^{x+y}$

$13 = Dec(a) = \underbrace{1 \cdot 2^3 + 1 \cdot 2^2}_{} + 0 \cdot 2^1 + 1 \cdot 2^0$

$$= 2^2 \left(1 \cdot \frac{2^3}{2^2} + 1 \cdot \frac{2^2}{2^2}\right) + 0 \cdot 2^1 + 1 \cdot 2^0$$

$$= 2^2 \underbrace{(1 \cdot 2^1 + 1 \cdot 2^0)}_{Dec(a^1)} + \underbrace{0 \cdot 2^1 + 1 \cdot 2^0}_{Dec(a^0)}$$

$$= 2^2 \cdot Dec(a^1) + Dec(a^0)$$

$$= 4 \cdot 3 + 1$$

$$ab = \underbrace{a^1 b^1}_{m_1} 2^{2\lfloor \frac{n}{2} \rfloor} + (\underbrace{a^1 b^0}_{m_2} + \underbrace{a^0 b^1}_{m_3}) 2^{\lfloor \frac{n}{2} \rfloor} + \underbrace{a^0 b^0}_{m_4}$$

$T(n)$ Mult (a, b)

$O(1)$ { If a & b are of const size, use traditional mult. algo & return $c = a \cdot b$

Compute a^1, a^0, b^1, b^0

$m_1 \leftarrow$ Mult (a^1, b^1) — $T(\frac{n}{2})$

$m_2 \leftarrow$ Mult (a^1, b^0) "

$m_3 \leftarrow$ Mult (a^0, b^1) "

$m_4 \leftarrow$ Mult (a^0, b^0) "

Return $m_1 \cdot 2^{2\lfloor \frac{n}{2} \rfloor} + (m_2 + m_3) \cdot 2^{\lfloor \frac{n}{2} \rfloor} + m_4$

$O(n)$ $O(n)$ $O(n)$ $O(n)$ $O(n)$

$$T(n) \leq c \quad \text{if} \quad n \leq 2$$

$$\leq cn + 3T(\tfrac{n}{2}) \quad O(n^2)$$

$O(n^{\log_2 3}) = O(n^{1.59})$

$m_2 + m_3 = a^1 b^0 + a^0 b^1$

$(a^1 + a^0)(b^1 + b^0)$

$= \underbrace{a^1 b^1}_{m_1} + \underbrace{a^1 b^0 + a^0 b^1}_{m_2 + m_3} + \underbrace{a^0 b^0}_{m_4}$

I/p: a^1, a^0, b^1, b^0

$m_1 = a^1 b^1$, $m_4 = a^0 b^0$