$\delta = \min\left(d(q_0, q_1), d(r_0, r_1)\right)$

∃ a $q \in Q$, $r \in R$ s.t. $d(q,r) < \delta$ iff
∃ $s \neq s' \in S$ s.t. $d(s,s') < \delta$

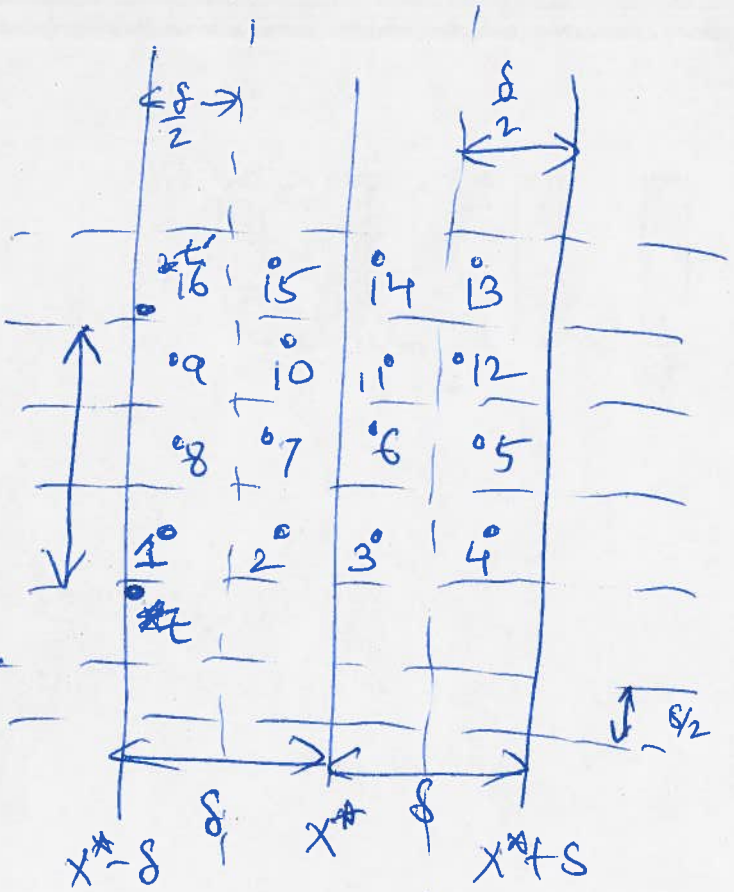
Recall: S_x, S_y from P_x & P_y in $O(n)$ time.

KICKASS PROPERTY LEMMA

If $s, s' \in S$ s.t. $d(s,s') < \delta \Rightarrow s$ & s' are within 15 positions of each other in S_y

$\Rightarrow O(n)$ time implementation for closest-in-box(...)
(Do a linear scan & keep track of closest pairs in $(i, i+1), (i, i+2), \ldots, (i, i+15)$)

Pf of Kickass Prop Lemma:



Assume t' is ≥ 16 positions away from t.

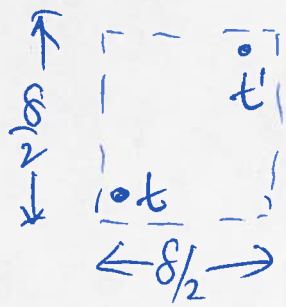
$\frac{3\delta}{2} > \delta$

$\Rightarrow$ contradiction!

t comes befor t' in S_y

$d(t,t') < \delta$

Claim 1:



t' & t can't be in the same box

$$d(t,t') \leq \sqrt{\left(\frac{\delta}{2}\right)^2 + \left(\frac{\delta}{2}\right)^2}$$

$$= \frac{\sqrt{2}}{2}\delta < \delta.$$

Contradiction as both $t,t' \in \mathbb{Q}$ or $t,t' \in \mathbb{R}$ $\Leftarrow$