O_j : optimal soln for $\{1, \dots, j\}$

$OPT(j)$ = value of O_j = $\sum_{i \in O_j} v_i$

Case 1 : $j \in O_j$

$O_j = j ; O_{p(j)}$ $\quad$ $OPT(j) = v_j + OPT(p(j))$

Case 2 : $j \notin O_j$

$O_j = O_{j-1}$ $\quad$ $OPT(j) = OPT(j-1)$

$$OPT(j) = \max\{ \overbrace{v_j + OPT(p(j))}^{1}, \overbrace{OPT(j-1)}^{2} \}$$

given $OPT(1), OPT(2), \dots, OPT(j-1)$.

By defn. $p(j) \leq j-1$

$\Rightarrow$ you know $OPT(p(j))$ & $OPT(j-1)$

1 $\geq$ 2 $\Rightarrow j \in O_j$

1 $<$ 2 $\Rightarrow j \notin O_j$

Ex: given $OPT(1) \dots OPT(n)$ $\Rightarrow$ can compute O_j.

Claim: If we compute $OPT(1), \dots, OPT(n)$

$\Rightarrow$ we can compute in $O(n)$ time $O = O_n$.