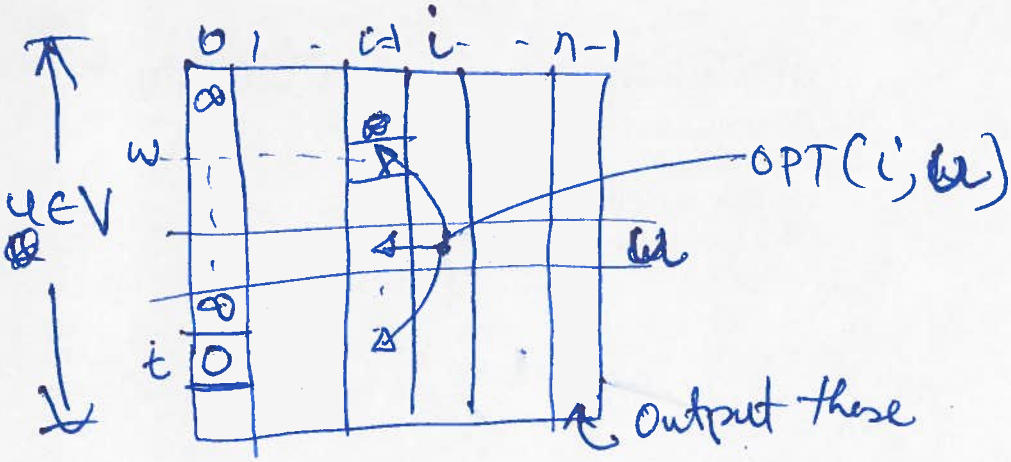
Bellman-Ford

$$OPT(i,u) = \min\Big(OPT(i-1,u)\ \cancel{+}\ \min_{(u,w)\in E} c_{u,w} + OPT(i-1,w)\Big)$$

To compute $OPT(i,u)$ what other values do you need to know.

Know $OPT(i-1,*)$ is enough to compute $OPT(i,*)$



Shortest-Path (s,t) $\quad [M : V \times \{0,..,n-1\}]$

0. $M[0,t]=0$, $M[0,u]=\infty$ $\forall u \neq t$ $[O(n)]$

1. For $i=0,1,..,n-1$ $\leftarrow n$

   For every $u \in V$ $\leftarrow n$

   $O(n) \rightarrow M[i,u] = \min\Big(M[i-1,u],\ \min_{(u,w)\in E} c_{u,w} + M[i-1,w]\Big)$

   $O(n^3)$

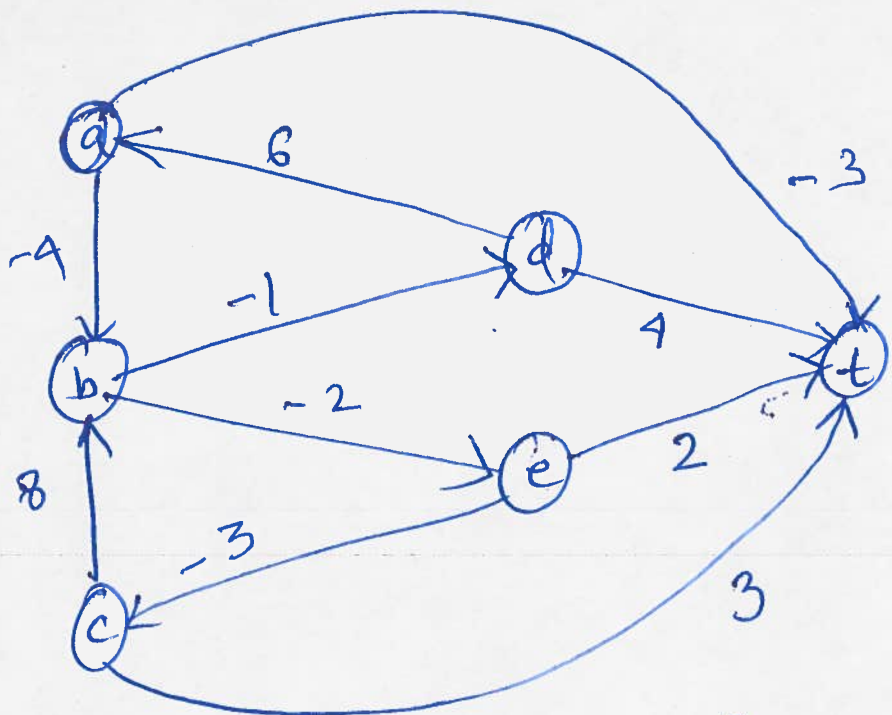
2. Output $M[n-1,s]$ $\forall s$.

   $O(n)$

Time: $\boxed{O(n^3)} \rightarrow \boxed{O(mn)}$

$\boxed{\text{Space } O(n^2) \rightarrow O(n)}$

Correctness: Induction



M[e,1]

M[b,2] $= \min(\infty, -2+2, -1+4) = 0$

	0	1	2	3	4	5
a	∞	-3				
b	∞	∞				
c	∞					
d	∞	4				
e	∞	2				
t	0					

OPT(d,5) = 0

$\mathrm{OPT}(a,1) = \min\left(\underset{\infty}{\mathrm{OPT}(a,0)}, \min\left(-4+\underset{\infty}{\mathrm{OPT}(b,0)}, \underset{0}{\mathrm{OPT}(t,0)}\right)\right)$

$\mathrm{OPT}(b,1) = \min(\infty, -2+\infty, -1+\infty) = \infty$