

② General case

→ Some engaged & some free

(Assume: Each woman proposes to each man $\leq$ once)

Let w be a free woman

$m \rightarrow$ the most preferred man w has
NOT proposed to

→ w proposes to m

Case 2.1: m is free

(m, w) get engaged

Case 2.2: (m, w') are engaged ($w' \neq w$)

Case 2.2.1: $w' > w$ in L_m

Nothing changes

Case 2.2.2: $w > w'$ in L_m

(m, w) get engaged. w' is free.

③ If all n men & n women are engaged, then all the pairs ~~get~~ married

Once

Obs 1) A man m gets engaged, he remains engaged + he keeps getting engaged to better women.

Obs 2) A woman w keeps getting engaged to worse men.

Q1) Does the algo terminate? (Efficiency)

Q2) Does the algo always output a stable matching?

Theorem 1: There are $\leq n^2$ iterations of the GS algo.

Pf idea: Notion of "Progress"

for iteration t , $P(t)$ s.t $\int \Rightarrow \# \text{ iterations} \leq U$.

(i) $P(t) \leq P(t) \leq U$

(ii) $P(t+1) \geq P(t)+1$