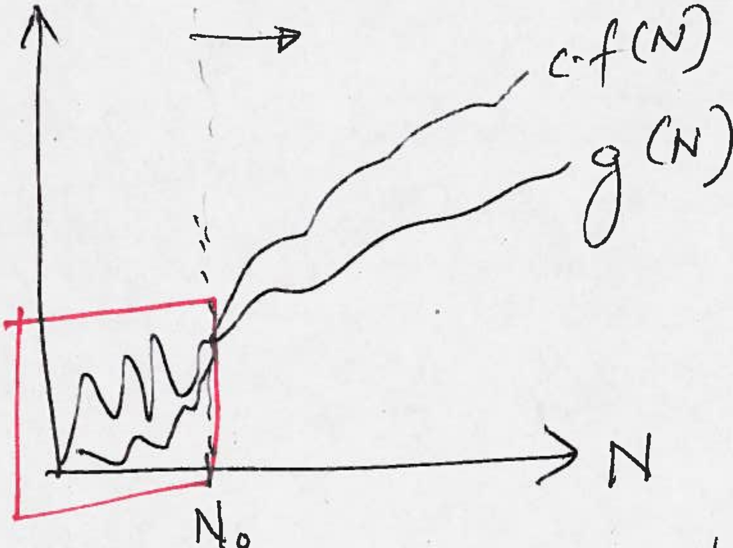
Def: $g(N)$ is $O(f(N))$ if $\exists$ a $N_0 \geq 0$ s.t.
(or g is $O(f)$) & a constant c

$\forall N \geq N_0$

$g(N) \leq c \cdot f(N)$



Q: $pN^2 + qN + r$

$O(N^2)$ $\leq qN^2$, $\leq r$

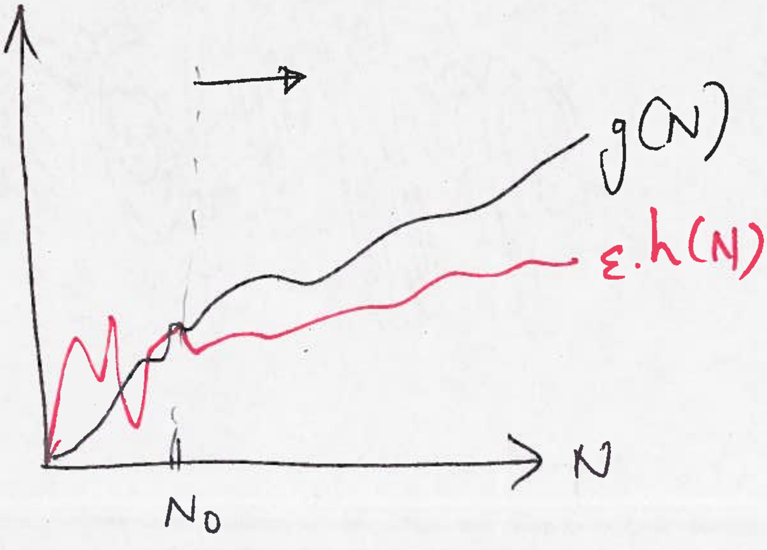
$\leq (p+q+r) N^2$

$\hookrightarrow O(N^3)$ ✓

$\hookrightarrow O(N)$ ✗

Def: $g(N)$ is $\Omega(h(N))$ if $\exists$ $N_0 \geq 0$
(g is $\Omega(h)$) & $\varepsilon > 0$ s.t.

$\forall N \geq N_0$ $g(N) \geq \varepsilon \cdot h(N)$



$pN^2 + qN + r$

$\Omega(N^2)$

$\varepsilon = p$, $N_0 \geq 0$

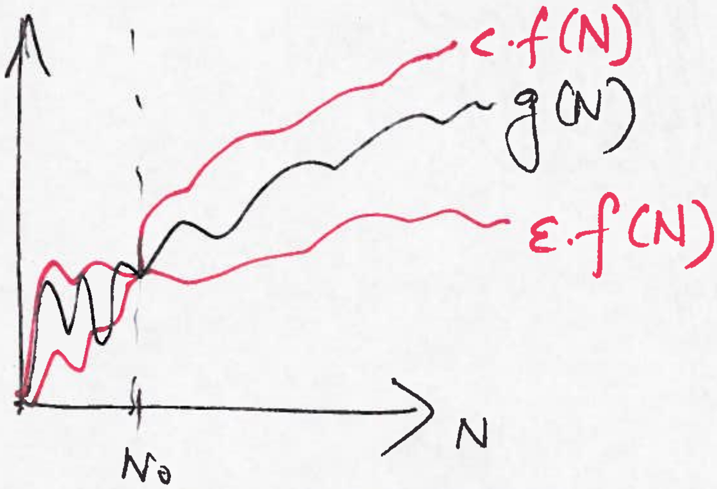
$\Omega(N^3)$ ✗

$\Omega(N)$ ✓

Def: $g(N)$ is $\Theta(f(N))$

if $g(N)$ is both $O(f(N))$

and $\Omega\, f(N)$



$pN^2 + qN + r$

$\rightarrow \Theta(N^2)$

$\rightarrow \Theta(N^3)$ ×

$\rightarrow \Theta(N)$ ×

Useful properties

① Transitivity g is $O(h)$ & h is $O(f) \Rightarrow g$ is $O(f)$

Similarly, Ω, Θ

② Sum of functions.

g is $O(h)$ & f is $O(h)$, $g+f$ is $O(h)$

$(f_1, \ldots, f_k$ are $O(h)$ $\quad f_1 + \cdots + f_k$ is $O(h))$

$k \rightarrow$ is constant

③ g is $O(h_1)$, f is $O(h_2)$

$g \cdot f$ is $O(h_1 \cdot h_2)$

also true for Ω & Θ

$T(n) = \max_{\substack{\text{all inputs } I \\ \text{of size } n}}$ steps taken by the algo on I.

<u>Case 1</u>: $T(n) \le U(n)$ [$O(U)$]

<u>Show</u>: $\forall$ inputs I of size n, # steps $\le U$. [$O(U)$]

$$\left(\equiv \max_{i=1..m} a_i \le U \right.$$

$$\Uparrow$$

$$\left. \forall i \ a_i \le U \right)$$

<u>Case 2</u>: $T(n) \ge L(n)$ [$\Omega(L)$]

$\exists$ an input I of size n # steps $\ge L$ [$\Omega(L)$]

$(\equiv \exists \ i \text{ s.t. } a_i \ge L)$

FIND $(a_1, \ldots, a_n; v)$ /* Output: i if $a_i = v$

-1 if no such i $\exists$.

runs $\le n$

① $\to$ for $(i = 1 .. n)$ $\rightsquigarrow$ #itr

② if $(a_i == v)$ return i. $\leftarrow O(1)$ $\}$ $\le c \cdot$ #itr

$\nwarrow c$

③ return -1. $\leftarrow O(1)$

$O(n \cdot 1) = O(n)$

$T(n) = O(n)$

step ①+② $\to O(n)$

step ③ $\to O(1) \le O(n)$

$T(n) = ($step ①+②$) +$ step 3

$= O(n) + O(n)$

$= O(n)$

true for all inputs of size $(n+1)$

$T(n) = \Omega(n)$

for any $n \ge 1$ Cons. $a_1 = n$

$a_2 = n - 1$

$\vdots$

$a_n = 1$

$v = 1$