Notes on Inheritance Networks

Stuart C. Shapiro

November 10, 2004

1 Introduction

These notes are derived from, and comment on Brachman & Levesque, *Knowledge Representation and Reasoning*, Chapter 10.

The topic is *defeasible inheritance.* Although, if interpreted strictly, the network is contradictory, the fix is to ignore a conclusion, but retain all hypotheses. (Compare belief revision.)

The Shortest Path Heuristic doesn't work in general.

2 Formal Account

2.1 Edges and Paths

Inheritance hierarchy: $\Gamma = \langle V, E \rangle$

I'll write edges in E as $a \longrightarrow x$, $a \longrightarrow\!\!\!\!\!+ \; x$, or $a \stackrel{?}{\longrightarrow} x$ and conclusions as $a \Longrightarrow x$ and $a \Longrightarrow\!\!\!\!\!+ \; x$.

Positive Path: $a \longrightarrow \cdots \longrightarrow x$ ($\geq$ 1 edge)

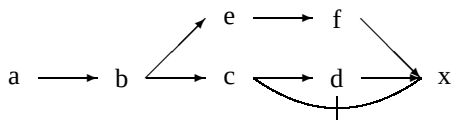
Negative Path: $a \longrightarrow \cdots \longrightarrow v \longrightarrow\!\!\!\!\!+ \; x$ ($\geq$ 1 edge, only last is negative.)

2.2 Support

A path (argument) *supports* a conclusion:

$a \longrightarrow \cdots \longrightarrow x$ supports $a \Longrightarrow x$
$a \longrightarrow \cdots \longrightarrow v \longrightarrow\!\!\!\!\!+ \; x$ supports $a \Longrightarrow\!\!\!\!\!+ \; x$

One conclusion may be supported by several arguments:



So defeasibility is about one argument defeating another argument.

Γ supports a path if the path is in Γ and the path is *admissible.*
Γ supports a conclusion if it supports a path that supports the conclusion.

2.3 Admissibility

A path

$$a \longrightarrow b \longrightarrow \cdots \longrightarrow v \dashrightarrow x$$

is admissible if every edge in it is admissible *with respect to (wrt)* a, its starting node.

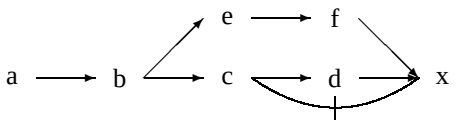
Edge $v \dashrightarrow x$ is admissible wrt a in the path

$$a \longrightarrow b \longrightarrow \cdots \longrightarrow v \overset{?}{\dashrightarrow} x$$

if there is a positive path p from a to v such that:

1. each edge in p is admissible wrt a;
2. no edge in p is *redundant* wrt a
3. no *node* in p is a preemptor of $v \dashrightarrow x$ wrt a.

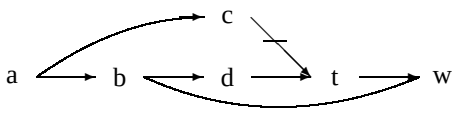
E.g., in



c is a preemptor of $d \longrightarrow x$ wrt a, so $d \longrightarrow x$ is not an admissible edge wrt a, and $a \longrightarrow b \longrightarrow c \longrightarrow d \longrightarrow x$ is not an admissible path. However, $a \longrightarrow b \longrightarrow e \longrightarrow f \longrightarrow x$ is an admissible path, and so is $a \longrightarrow b \longrightarrow c \dashrightarrow x$.

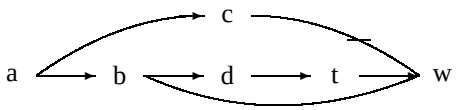
2.4 Redundancy

Besides the obvious, in



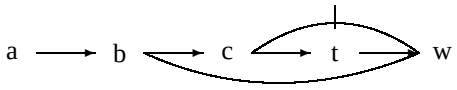
$b \longrightarrow w$ is not redundant, because without it, $a \Longrightarrow t$ is controversial, and therefore, so is $a \Longrightarrow w$.

According to the text, in



$b \longrightarrow w$ is also not redundant, but I don't see why.

I suspect that what was meant was



because c is a preemptor of $t \longrightarrow w$ wrt a.

2.5 Extensions

In general, an *extension* of a KB is a maximally consistent deductive closure of the KB. If a KB is inconsistent, it will have several extensions.

Γ is *a-connected* iff there is a path (positive or negative) from a to every node, x, in Γ.

Γ is (potentially) *ambiguous* wrt a at x if there is both a positive and a negative path from a to x.

A *credulous extension* of Γ wrt a is a maximal unambiguous a-connected subhierarchy of Γ wrt a.

If X and Y are two credulous extensions of Γ wrt a, X is *preferred* to Y iff there is some v such that they agree on all paths from a to v, but there is an edge $v \stackrel{?}{\longrightarrow} x$ that is: inadmissible in Γ; in Y; but not in X.

A credulous extension is a *preferred extension* if there is no other credulous extension that is preferred to it.

2.6 Reasoning Styles

credulous reasoning: Choose any preferred extension, and believe all the conclusions supported by it.

skeptical reasoning: Believe only the conclusions supported by paths that are present in all preferred extensions.

ideal skeptical reasoning: Believe only the conclusions that are supported by every preferred extension.

A credulous/skeptical/ideally-skeptical reasoner is one that uses that style of reasoning.

2.6.1 Example of difference between skeptical and ideally-skeptical reasoners

Question: Give pairs of employees s.t. one earns more than the other.

KB1: John earns $30,000; Mary earns $50,000.

KB2: John earns $35,000; Mary earns $55,000.