Problem 2

1. Build a decision tree:
The entropy equation is:

$$E = -\sum_j P(w_j) \log P(w_j)$$

According to the data, we have the total labels 9+, 4-. So the entropy for the whole data is:

$$E = -\frac{9}{13} \log \frac{9}{13} - \frac{4}{13} \log \frac{4}{13} = 0.8905$$

If we classify the data with Wakeup:
Early: 4+

$$E = -\frac{4}{4} \log \frac{4}{4} = 0$$

Normal: 2+, 3-

$$E = -\frac{2}{5} \log \frac{2}{5} - \frac{3}{5} \log \frac{3}{5} = 0.9710$$

Late: 3+, 1-

$$E = -\frac{3}{4} \log \frac{3}{4} - \frac{1}{4} \log \frac{1}{4} = 0.8113$$

$$Gain(Wakeup) = 0.8905 - \frac{4}{13} \cdot 0 - \frac{5}{13} \cdot 0.9710 - \frac{4}{13} \cdot 0.8113 = 0.2674$$

If we classify the data with HaveTalk:
Yes: 6+, 1-

$$E = -\frac{6}{7}\log\frac{6}{7} - \frac{1}{7}\log\frac{1}{7} = 0.5917$$

No: 3+, 3-

$$E = -\frac{3}{6}\log\frac{3}{6} - \frac{3}{6}\log\frac{3}{6} = 1$$

$$Gain(HaveTalk) = 0.8905 - \frac{7}{13}\cdot 0.5917 - \frac{6}{13}\cdot 1 = 0.1104$$

If we classify the data with Weather:

Sunny: 6+, 2-

$$E = -\frac{6}{8}\log\frac{6}{8} - \frac{2}{8}\log\frac{2}{8} = 0.8113$$

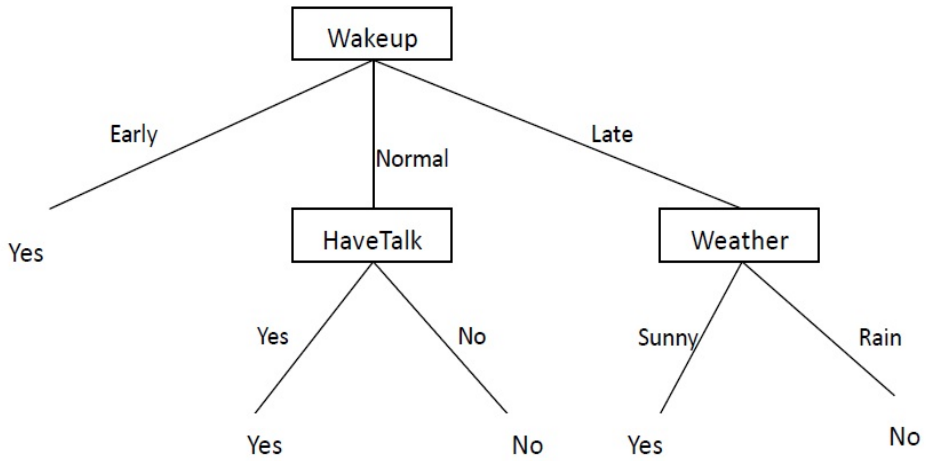
Rain: 3+, 2-

$$E = -\frac{3}{5}\log\frac{3}{5} - \frac{2}{5}\log\frac{2}{5} = 0.9710$$

$$Gain(Weather) = 0.8905 - \frac{8}{13}\cdot 0.8113 - \frac{5}{13}\cdot 0.9710 = 0.0178$$

Since $Gain(Wakeup)$ is the largest one, we choose Wakeup in this step. The following steps are skiped since they are quite similar to this one.

The final decision tree is:



2. According to the tree learned, the sample should be classified to NO.

3. Yes, the sample can be classified with missing data: ***marginalize***.
If Wakeup = Early, definitely the student will go to school.
If Wakeup = Normal, since HaveTalk = Yes, the student will go to school.
If Wakeup = Late, since Weather = Sunny, the student will go to school.
So for this sample, no matter what the wake up time is, the student will go to school.