Syntactic Methods (Strings and Grammars)

Jason Corso

SUNY at Buffalo

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- Notation
  - Assume each discrete character is taken from an alphabet $\mathcal{A}$.
  - Use the same vector notation for a string: $\mathbf{x} =$"AGCTTC".
  - Call a particularly long string *text*.
  - Call a contiguous substring of $\mathbf{x}$ a *factor*.

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- **String Matching with the "Don't–Care" Symbol**: This is the same as basic string matching, but with the special symbol–$\emptyset$, the *don't care* symbol–which can match any other symbol.

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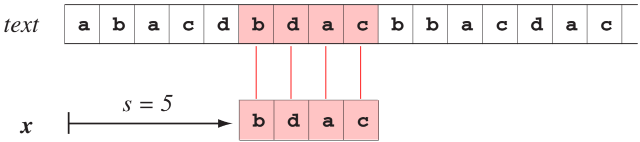

Naive String Matching

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begin initialize A, x, n ← |text|,  m ← |x|
  s ← 0
  while s ≤ n − m
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- The weakness comes from the fact that it does not use any information about a potential shift s to compute the next possible one s.

Boyer-Moore String Matching

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begin initialize 𝒜, x, n ← |text|, m ← |x|
  s ← 0
  ℱ(x) ← last-occurrence function
  𝒢(x) ← good-suffix function
  while s ≤ n − m
    j ← m
    while j > 0 and x[j] = text[s + j]
      j ← j − 1
      if j = 0
        then print "pattern occurs at shift" s
             s ← s + 𝒢(0)
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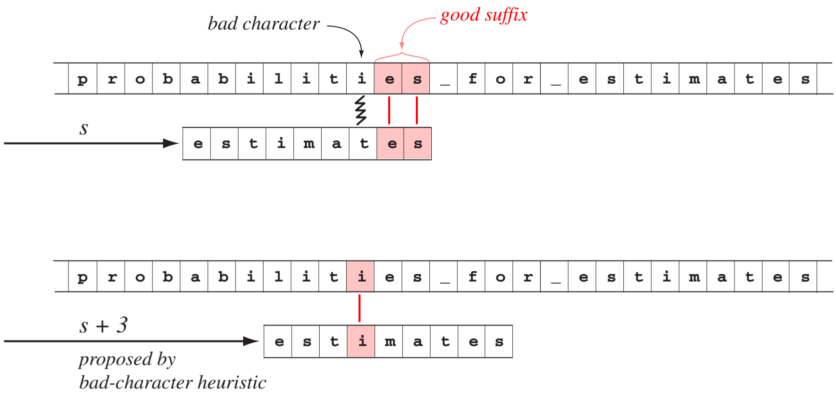
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good suffix
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s
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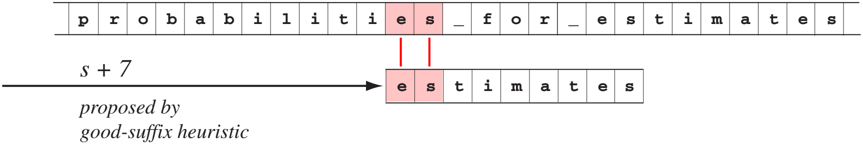

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- These heuristics make the Boyer-Moore string searching algorithm one of the most attactive string-matching algorithms on serial computers.

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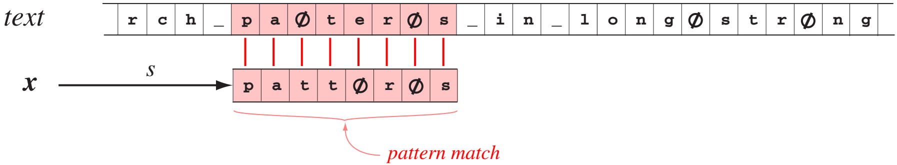

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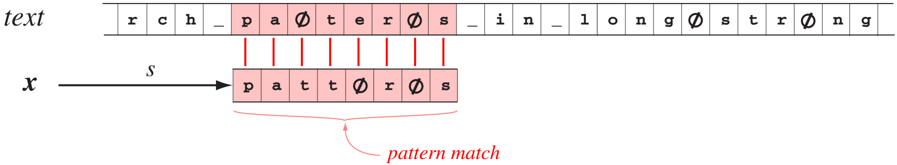

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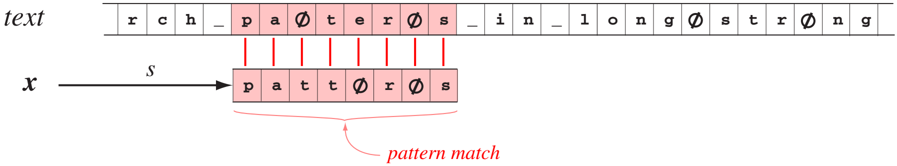
- Extending Boyer-Moore is quite a challenge...



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- But, how do we compute the distance between two strings?
- **Edit distance** is a possibility, which describes how many fundamental operations are required to transform $\mathbf{x}$ into $\mathbf{y}$, another string.

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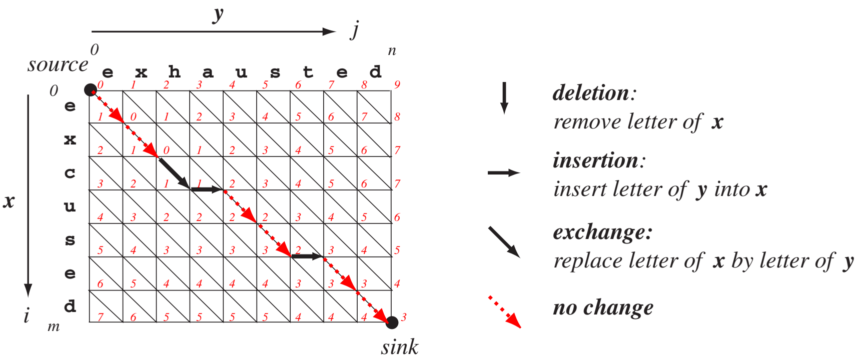
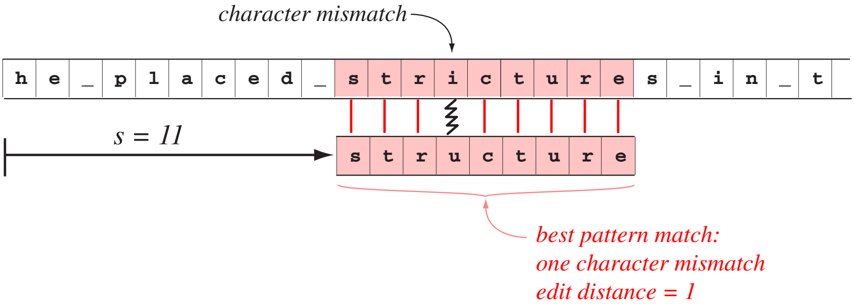
3. **Deletions**: a character in $\mathbf{x}$ is deleted, thereby decreasing the length of $\mathbf{x}$ by one character.
4. **Transpositions**: two neighboring characters in $\mathbf{x}$ change positions. But, this is not really a fundamental operation because we can always encode it by two substitutions.

The basic Edit Distance algorithm builds an $m \times n$ matrix of costs and uses it to compute the distance. Below is a graphic describing the basic idea. For more details read section 8.5.2 on your own.



String Matching with Errors

- Problem: Given a pattern $\mathbf{x}$ and *text*, find the shift for which the edit distance between $\mathbf{x}$ and a factor of *text* is minimum.
- Proceed in a similar manner to the Edit Distance algorithm, but need to compute a second matrix of minimum edit values across the rows and columns.



String Matching Round-Up

- We've covered the basics of string matching.
- How does these methods relate to the temporal ones we saw last week?
- While learning has found general use in pattern recognition, its application in basic string matching has been quite limited.

Grammatical Methods

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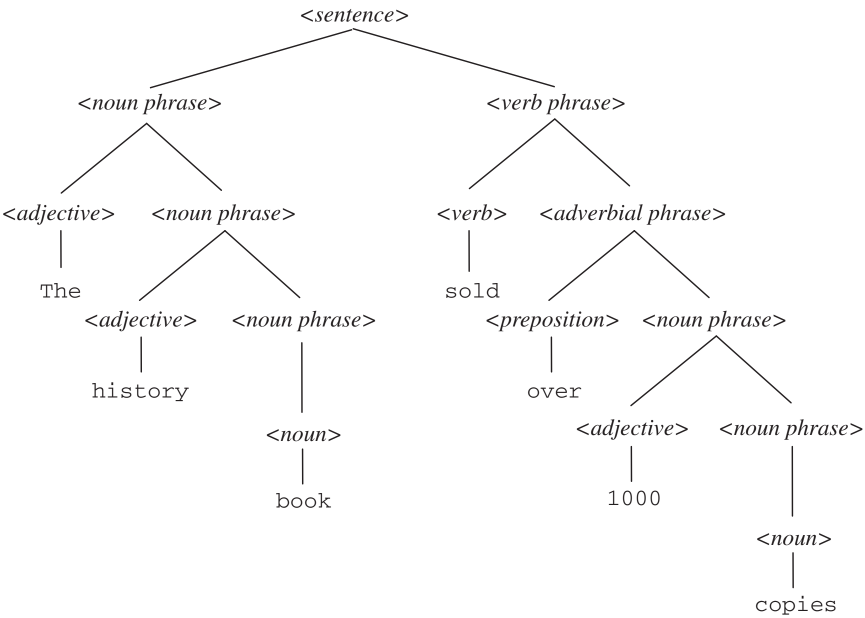
Grammatical Methods

- The earlier discussion on string matching paid no attention to any models that might have underlied the creation of the sequence of characters in the string.
- In the case of grammatical methods, we are concerned with the set of rules that were used to generate the strings.
- In this case, the structure of the strings is fundamental. And, the structure is often **hierarchical**.



<sentence>
<noun phrase>
<verb phrase>
<adjective>
<noun phrase>
<verb>
<adverbial phrase>
The
sold
<adjective>
<noun phrase>
<preposition>
<noun phrase>
history
over
<adjective>
<noun phrase>
<noun>
1000
book
<noun>
copies

Grammars

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- Thus, we denote a grammar by $G = (\mathcal{A}, \mathcal{I}, \mathcal{S}, \mathcal{P})$.
- The **language** generated by a grammar, $\mathcal{L}(G)$, is the set of all strings (possibly infinite) that can be generated by G.

Consider an abstract example:

- Let $\mathcal{A} = \{$a,b,c$\}$.
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---

root	S

Consider an abstract example:

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---

root	S
$\mathbf{p}_1$	aBA

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root	S
$\mathbf{p}_1$	aBA
$\mathbf{p}_6$	abA

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root	S
$\mathbf{p}_1$	aBA
$\mathbf{p}_6$	abA
$\mathbf{p}_4$	abc

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$\mathbf{p}_1$	aBA
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root	S
$\mathbf{p}_1$	aBA
$\mathbf{p}_6$	abA
$\mathbf{p}_4$	abc

root	S
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root	S
$\mathbf{p}_1$	aBA
$\mathbf{p}_6$	abA
$\mathbf{p}_4$	abc

root	S
$\mathbf{p}_1$	aSBA
$\mathbf{p}_1$	aaBABA

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---

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$\mathbf{p}_1$	aBA
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$\mathbf{p}_4$	abc

root	S
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$\mathbf{p}_6$	aabABA
$\mathbf{p}_2$	aabBAA

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$\mathbf{p}_1$	aBA
$\mathbf{p}_6$	abA
$\mathbf{p}_4$	abc

root	S
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$\mathbf{p}_2$	aabBAA
$\mathbf{p}_3$	aabbAA

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---

root	S
$\mathbf{p}_1$	aBA
$\mathbf{p}_6$	abA
$\mathbf{p}_4$	abc

root	S
$\mathbf{p}_1$	aSBA
$\mathbf{p}_1$	aaBABA
$\mathbf{p}_6$	aabABA
$\mathbf{p}_2$	aabBAA
$\mathbf{p}_3$	aabbAA
$\mathbf{p}_4$	aabbcA

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root	S
$\mathbf{p}_1$	aBA
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root	S
$\mathbf{p}_1$	aSBA
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$\mathbf{p}_1$	aBA
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$\mathbf{p}_1$	aaBABA
$\mathbf{p}_6$	aabABA
$\mathbf{p}_2$	aabBAA
$\mathbf{p}_3$	aabbAA
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These are two examples of **productions**.

Another example...English.

- The alphabet is all English words:
  $\mathcal{A} = \{$the, history, book, sold, over, ...$\}$.

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- The root symbol is $\mathcal{S} = \{\langle\text{sentence}\rangle\}$.
- A restricted set of production rules is

$$\mathcal{P} = \left\{ \begin{array}{rcl} \langle\text{sentence}\rangle & \rightarrow & \langle\text{noun phrase}\rangle\langle\text{verb phrase}\rangle \\ \langle\text{noun phrase}\rangle & \rightarrow & \langle\text{adjective}\rangle\langle\text{noun phrase}\rangle \\ \langle\text{verb phrase}\rangle & \rightarrow & \langle\text{verb phrase}\rangle\langle\text{adverb phrase}\rangle \\ \langle\text{noun}\rangle & \rightarrow & \text{book OR theorem OR}\dots \\ \langle\text{verb}\rangle & \rightarrow & \text{describes OR buys OR}\dots \\ \langle\text{adverb}\rangle & \rightarrow & \text{over OR frankly OR}\dots \end{array} \right\}$$

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- Of course, this subset of the rules for English grammar does not prevent the generation of meaningless sentences like *Squishy green dreams hop heuristically*.

Types of String Grammars

- **Type 0: Unrestricted or Free**. There are no restrictions on the production rules and thus there will be no constraints on the strings they can produce.
  - These have found little use in pattern recognition because so little information is provided when one knows a particular string has come from a Type 0 grammar, and learning can be expensive.

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  - These have found little use in pattern recognition because so little information is provided when one knows a particular string has come from a Type 0 grammar, and learning can be expensive.
- **Type 1: Context-Sensitive**. A grammar is called context-sensitive if every rewrite rule is of the form

$$\alpha I \beta \rightarrow \alpha x \beta \tag{1}$$

where both α and β are any strings of intermediate or terminal symbols, I is an intermediate symbol, and x is an intermediate or terminal symbol.

- **Type 2: Context-Free**. A grammar is called context-free if every production rule is of the form

$$I \to x \tag{2}$$

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where I is an intermediate symbol and x is an intermediate or terminal symbol.

- Any context free grammar can be converted into one in **Chomsky normal form** (CNF), which has rules of the form:

$$A \rightarrow BC \qquad \text{and} \qquad A \rightarrow z \tag{3}$$

where A, B, C are intermediate symbols and z is a terminal symbol.

- **Type 3: Finite State of Regular**. A grammar is called regular if every production rule is of the form

$$\alpha \rightarrow z\beta \qquad \text{OR} \qquad \alpha \rightarrow z \tag{4}$$

where α and β are made up of intermediate symbols and z is a terminal symbol.

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- These grammars can be generated by a finite state machine.

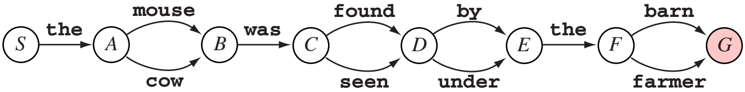

FIGURE 8.16. One type of finite-state machine consists of nodes that can emit terminal symbols ("`the`," "`mouse`," etc.) and transition to another node. Such operation can be described by a grammar. For instance, the rewrite rules for this finite-state machine include $S \to$ `the`A, $A \to$ `mouse`B *OR* `cow`B, and so on. Clearly these rules imply this finite-state machine implements a type 3 grammar. The final internal node (shaded) would lead to the null symbol ϵ. From: Richard O. Duda, Peter E. Hart, and David G. Stork, *Pattern Classification*. Copyright © 2001 by John Wiley & Sons, Inc.

Parsing – Recognition with Grammars

- Given a test sentence, $\mathbf{x}$, and c grammars, $G_1, G_2, \ldots, G_c$, we want to classify the test sentence according to which grammar could have produced it.

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- **Parsing** is the process of finding a derivation in a grammar G that leads to $\mathbf{x}$, which is quite more difficult than directly forming a derivation.
- **Bottom-Up Parsing** starts with the test sentence $\mathbf{x}$ and seeks to simplify it so as to represent it as the root symbol.

Parsing – Recognition with Grammars

- Given a test sentence, $\mathbf{x}$, and c grammars, $G_1, G_2, \ldots, G_c$, we want to classify the test sentence according to which grammar could have produced it.
- **Parsing** is the process of finding a derivation in a grammar G that leads to $\mathbf{x}$, which is quite more difficult than directly forming a derivation.
- **Bottom-Up Parsing** starts with the test sentence $\mathbf{x}$ and seeks to simplify it so as to represent it as the root symbol.
- **Top-Down Parsing** starts with the root node and successively applies productions from $\mathcal{P}$ with the goal of finding a derivation of the test sentence $\mathbf{x}$.

Bottom-Up Parsing

- The basic approach is to use candidate productions from $\mathcal{P}$ "backwards", which means we want to find the rules whose right hand side matches part of the current string. Then, we replace that part with a segment that could have produced it.
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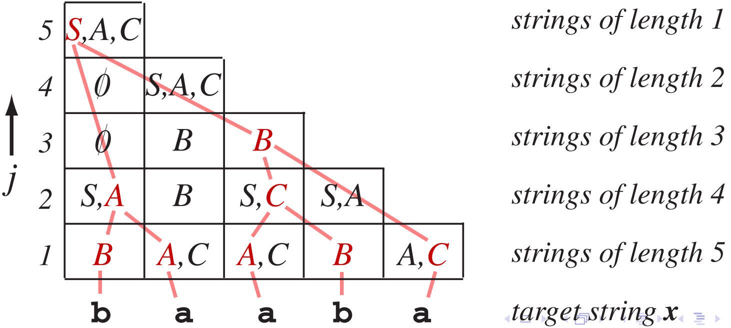
- Denote the individual terminal characters in the string to be parsed as x_i for $i = 1, \ldots, n$.

- Consider an example grammar G with two terminal symbols, $\mathcal{A} = \{a, b\}$, three intermediate symbols, $\mathcal{I} = \{A, B, C\}$, the root symbol S, and four production rules,

$$\mathcal{P} = \left\{ \begin{array}{lll} \mathbf{p}_1 : & S \rightarrow AB \text{ OR } BC \\ \mathbf{p}_2 : & A \rightarrow BA \text{ OR } a \\ \mathbf{p}_3 : & B \rightarrow CC \text{ OR } b \\ \mathbf{p}_4 : & C \rightarrow AB \text{ OR } a \end{array} \right\} .$$

- The following is the parse table for the string $\mathbf{x} =$ "baaba".



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- We will not cover grammar inference, learning the grammar.