

# Abstraction in PyLith

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PyLith...

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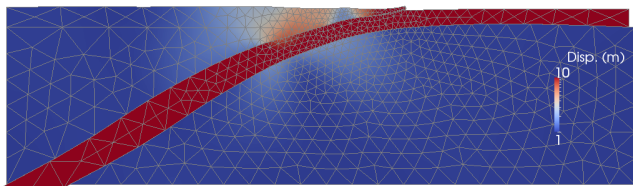
simulates crustal deformation  
focused on earthquake faulting,  
both quasi-static and dynamic.

PyLith...

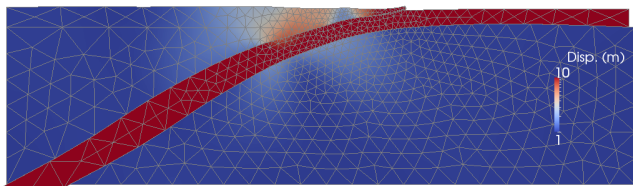
is actively developed since 2004,  
with >1000 downloads per release.

PyLith...

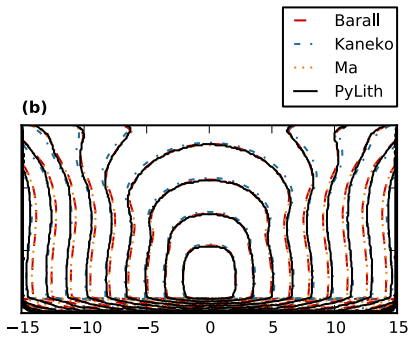
has over 180 citations (Scholar),  
and 60 papers based on it (CIG).

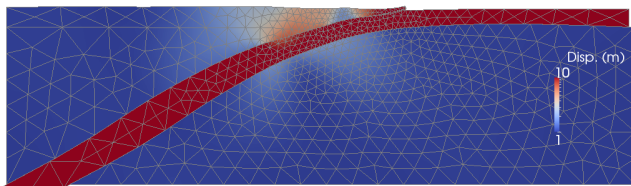


PyLith supports...  
nonlinear fault rheologies,



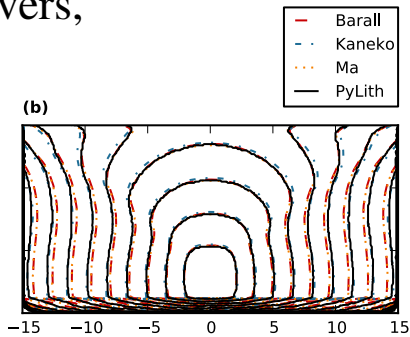
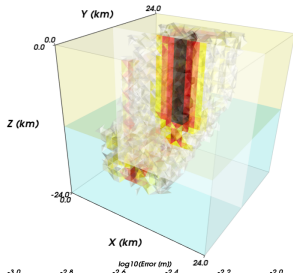
PyLith supports...  
nonlinear fault rheologies,  
flexible rupture definitions,

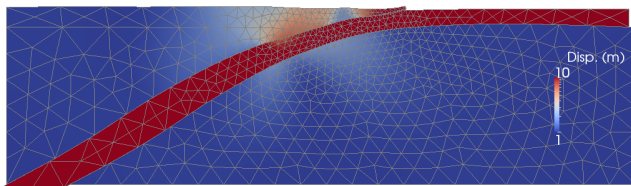




PyLith supports...

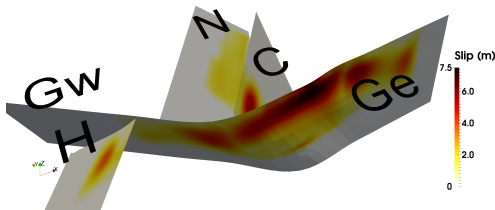
- nonlinear fault rheologies,
- flexible rupture definitions,
- scalable FEM solvers,





PyLith supports...

- nonlinear fault rheologies,
- flexible rupture definitions,
- scalable FEM solvers,
- and output over subsurfaces.



PyLith 3 will support...

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- multiphysics problems,
- geometric MG on the full problem,
- and anisotropic AMR.

We want to enhance **functionality**,  
while limiting **complexity**.

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while limiting **complexity**.

We achieve this goal through flexible,  
generalizable interfaces.

# Abstractions

Equations

Functions

Adaptation

In Pylith 3, you specify equations pointwise.

$$\begin{aligned} F(u) = & \int_{\Omega} \psi \cdot f_0(u, \nabla u, x) + \\ & \int_{\Omega} \nabla \psi : \vec{f}_1(u, \nabla u, x) + \\ & \sum_{\Gamma} \int_{\Gamma} \psi^{bd} \cdot f_0^{bd}(u, \nabla u, x) + \\ & \sum_{\Gamma} \int_{\Gamma} \nabla \psi^{bd} : \vec{f}_1^{bd}(u, \nabla u, x) \end{aligned}$$

In Pylith 3, you specify equations pointwise.

## Linear Isotropic Elasticity ( $u; \lambda, \mu$ )

---

```
f1_u (...) {  
    for (c = 0; c < Nc; ++c) {  
        for (d = 0; d < dim; ++d) {  
            f1[c*dim+d] += mu*(u_x[c*dim+d] + u_x[d*dim+c]);  
            f1[c*dim+c] += lambda*u_x[d*dim+d];  
        }  
    }  
}
```

---

In Pylith 3, you specify equations pointwise.

## Large Deformation Elasticity ( $u, p; \kappa, \mu$ )

---

```
f1_u (...) {
    Cof3D(cofu_x, u_x); Det3D(&detu_x, u_x);
    p = u[uOff[1]] + kappa * (detu_x - 1.0);
    for (c = 0; c < Nc; ++c) {
        for (d = 0; d < dim; ++d) {
            f1[c*dim+d] = mu * u_x[c*dim+d] + p * cofu_x[c*dim+d];
        }
    }
}

f0_p (...) {
    Det3D(&detu_x, u_x);
    f0[0] = detu_x - 1.0;
}

f0_bd_u(..., n) {
    const PetscScalar wall_p = a[aOff[1]];
    Cof3D(cofu_x, u_x);
    for (c = 0; c < Nc; ++c) {
        for (d = 0; d < dim; ++d) f0[c] += cofu_x[c*dim+d] * n[d];
        f0[c] *= wall_p;
    }
}
```

---

In Pylith 3, you specify equations pointwise.

## Poroelasticity ( $u, e, p_f; \lambda, \mu, \alpha, \kappa$ )

---

```
f1_u (...) {  
    p_f = u[uOff[2]];  
    for (c = 0; c < Nc; ++c) {  
        for (d = 0; d < dim; ++d) {  
            f1[c*dim+d] += mu*(u_x[c*dim+d] + u_x[d*dim+c]);  
            f1[c*dim+c] += lambda*u_x[d*dim+d];  
        }  
        f1[c*dim+c] += alpha*p_f;  
    }  
}  
  
f0_e (...) {  
    for (d = 0; d < dim; ++d) divu += u_x[d*dim+d];  
    f0[0] = divu - u[uOff[1]];  
}  
  
f1_p (...) {  
    for (d = 0; d < dim; ++d) f0[d] -= kappa*u_x[uOff_x[2]+d];  
}
```

---

# Abstractions

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In Pylith 3, finite element solutions and functions are interchangeable.

Import data from function  $f$  into FEM vector  $A$

---

```
DMplexProject(..., f, ..., A)
```

---

This works in any dimension.

In Pylith 3, finite element solutions and functions are interchangeable.

Import data from function  $f$  into FEM vector  $A$

PyLith's `spatialdata` can be used as  $f$ , and evaluated at any  $x$ , independent of the mesh.

In Pylith 3, finite element solutions and functions are interchangeable.

Create a boundary condition  $C$  from a function  $f$  and constitutive model parameters in vector  $A$

---

```
DMPlexProject(..., f, ..., A, C)
```

---

`spatialdata` makes boundary/initial condition independent of the mesh.

In Pylith 3, finite element solutions and functions are interchangeable.

Create a boundary condition  $C$  from a function  $f$  and *boundary* model parameters in vector  $A$ ,

---

```
DMplexProject (... , f, ... , A, C)
```

---

such as time-dependent boundary displacements.

In Pylith 3, finite element solutions and functions are interchangeable.

Create a rupture condition  $F$  from a function  $f$  and *boundary* model parameters in vector  $A$ ,

---

```
DMplexProject (... , f, ... , A, F)
```

---

such as rupture parameters that sequence events.

In Pylith 3, finite element solutions and functions are interchangeable.

Output normal stress on fault  $S$  from a function  $s$  and *bulk* model solution in vector  $A$

---

```
DMPlexProject (... , s , ... , A, S)
```

```
s (... ) {  
    for (c = 0; c < Nc; ++c) {  
        for (d = 0; d < dim; ++d) {  
            stress[c*dim+d] += mu*(u_x[c*dim+d] + u_x[d*dim+c]);  
            stress[c*dim+c] += lambda*u_x[d*dim+d];  
        }  
        for (d = 0; d < dim; ++d) f0[c] += stress[c*dim+d]*n[d];  
    }  
}
```

---

In Pylith 3, finite element solutions and functions are interchangeable.

Update PyLith state variables  $SV$  with function  $sv$ , using the old state variables and old solution as auxiliary data  $A$ ,

---

`DMProject (... , sv , ... , A, SV)`

---

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Equations

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In Pylith 3, you can adapt anisotropically  
around fault surfaces.

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Since we have abstracted

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boundary conditions

fault conditions

discretizations

away from the mesh, we can easily form  
new problems on adapted meshes.

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around fault surfaces.

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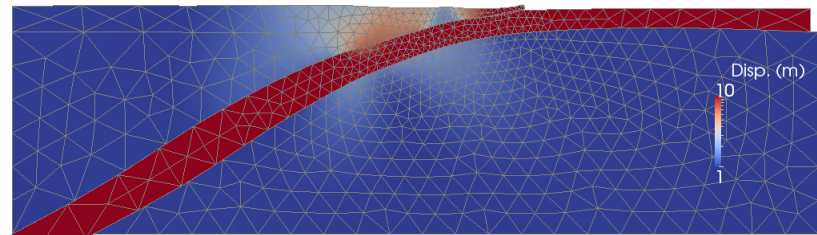
fault conditions

discretizations

away from the mesh, we can easily form  
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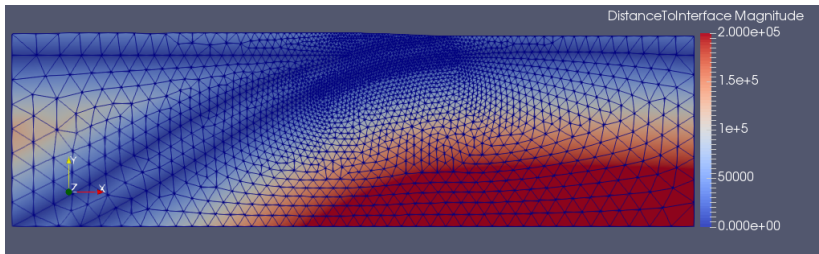
This also enables MG on the full problem.

In Pylith 3, you can adapt anisotropically around fault surfaces.



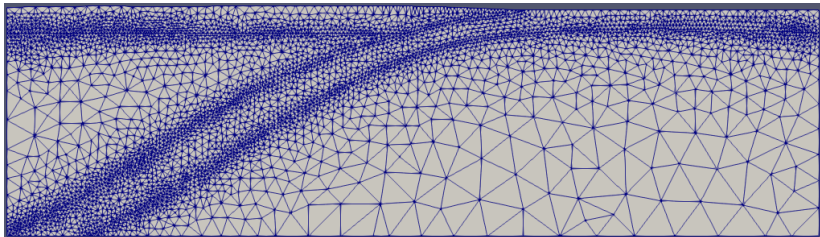
2D subduction zone

In Pylith 3, you can adapt anisotropically around fault surfaces.



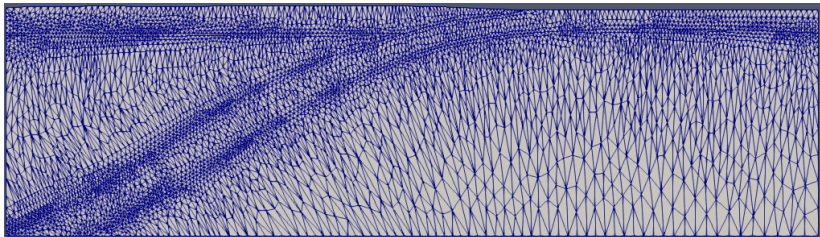
2D subduction zone

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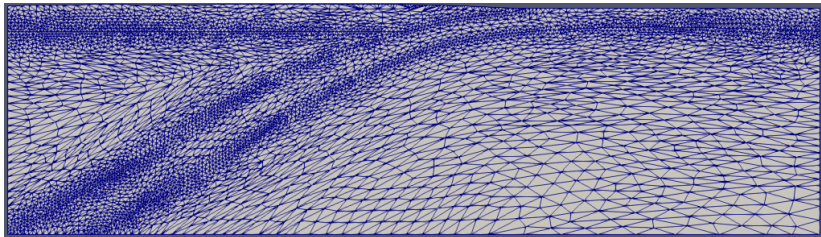
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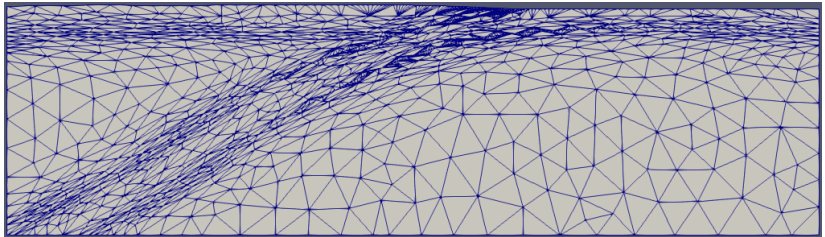
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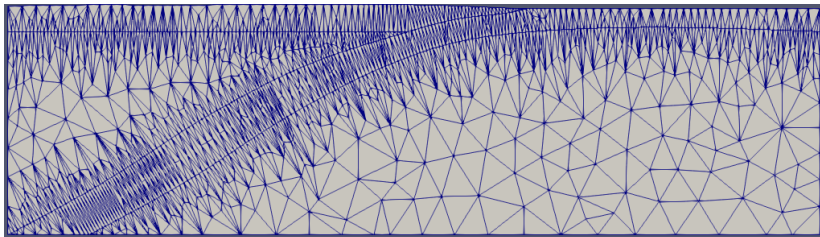
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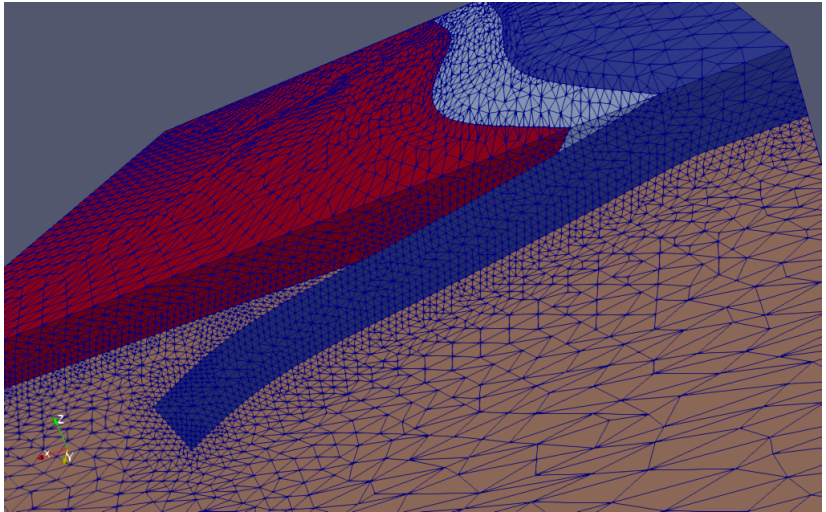
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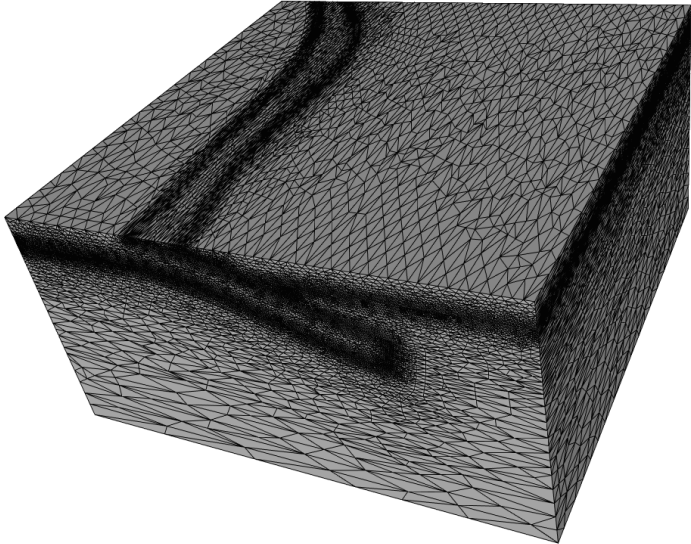
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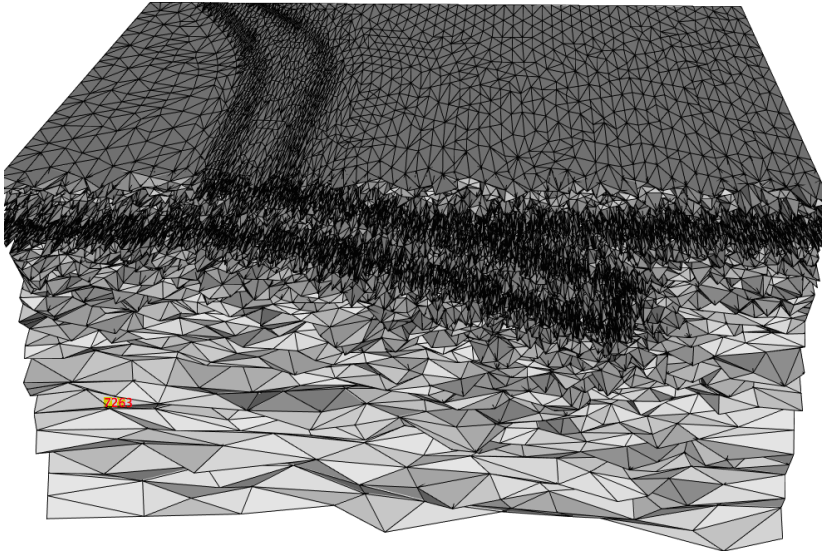
3D subduction zone

In Pylith 3, you can adapt anisotropically around fault surfaces.



3D subduction zone

In Pylith 3, you can adapt anisotropically around fault surfaces.



3D subduction zone

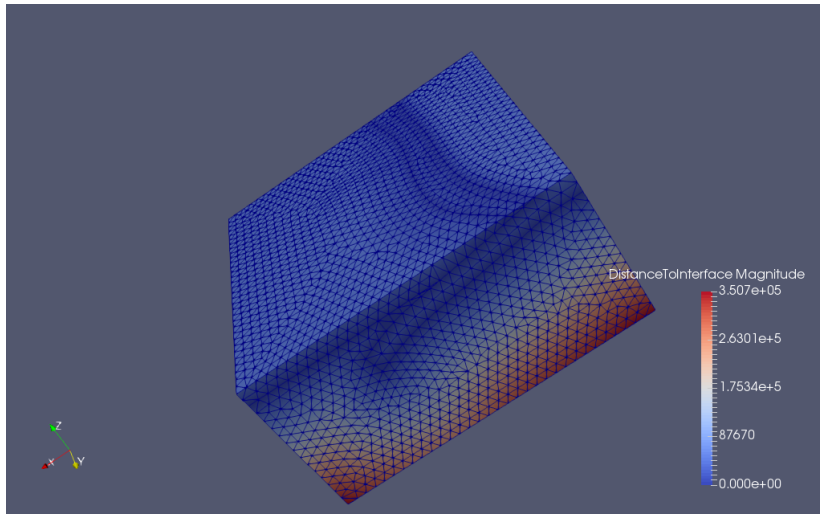
# PyLith

<https://github.com/geodynamics/pylith/>

<https://geodynamics.org/cig/software/pylith/>

[cig-short@geodynamics.org](mailto:cig-short@geodynamics.org)

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