How to Choose an Algorithm

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RELACS People

How do we choose an algorithm?

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We choose the fastest one…

Timing is tricky. It’s sensitive to

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machine characteristics

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problem details

Proxy measures can simplify design:

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Computation (HPL)

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Bandwidth (Roofline)

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Computation	(HPL)
Bandwidth	(Roofline)
Latency	(LogP)

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Computation	(HPL)
Bandwidth	(Roofline)
Latency	(LogP)
Concurrency	

These models can answer. . .

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Does this implementation scale weakly?

These models can answer. . .

Does this implementation scale weakly? strongly?

These models can answer…

Is one implementation more efficient than another on this machine?

What about questions like...

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Should I discretize this problem with CG or DG?

What about questions like...

Should I solve using the Picard or Newton Method?

The key notion we are missing is

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accuracy

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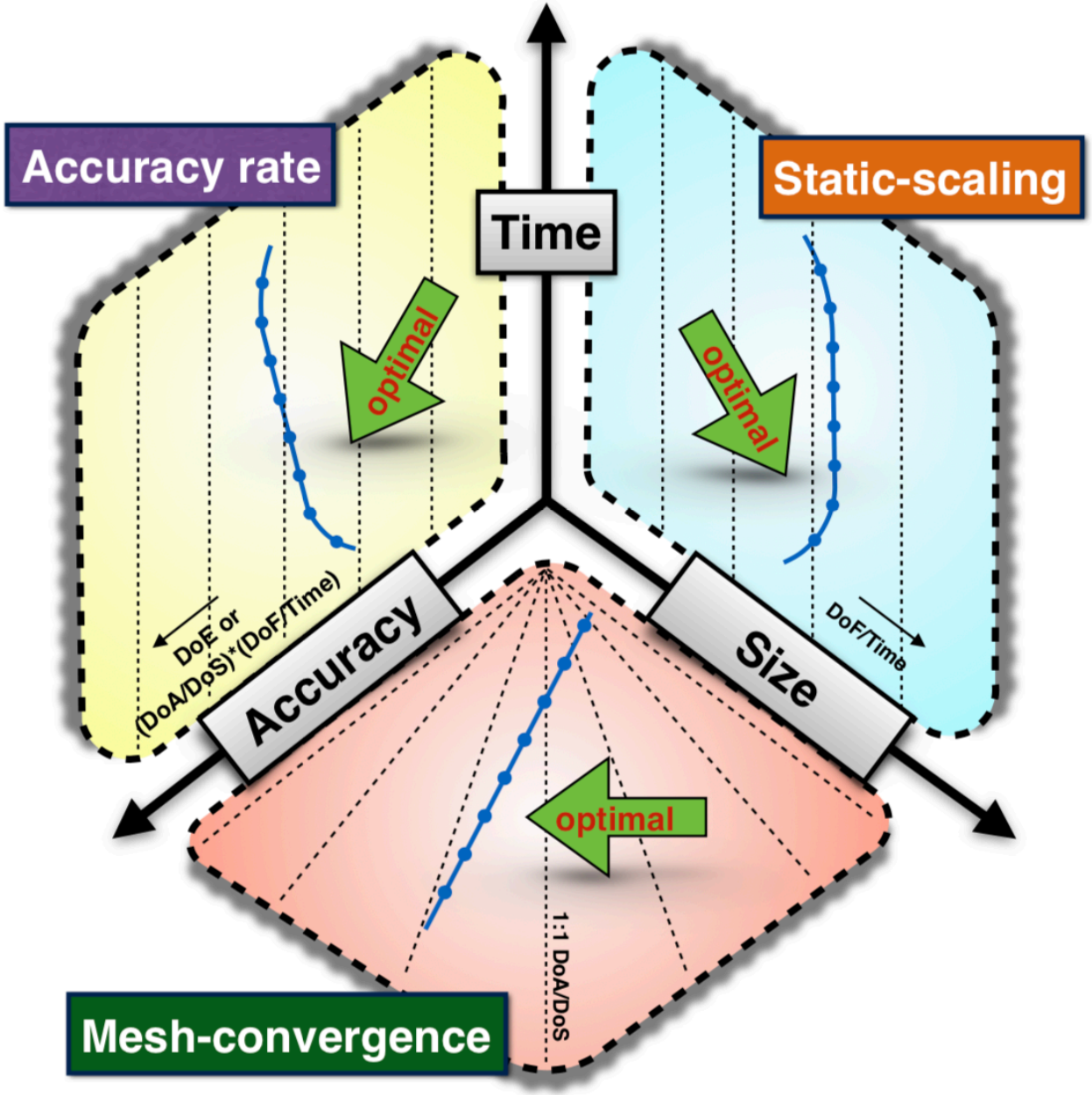
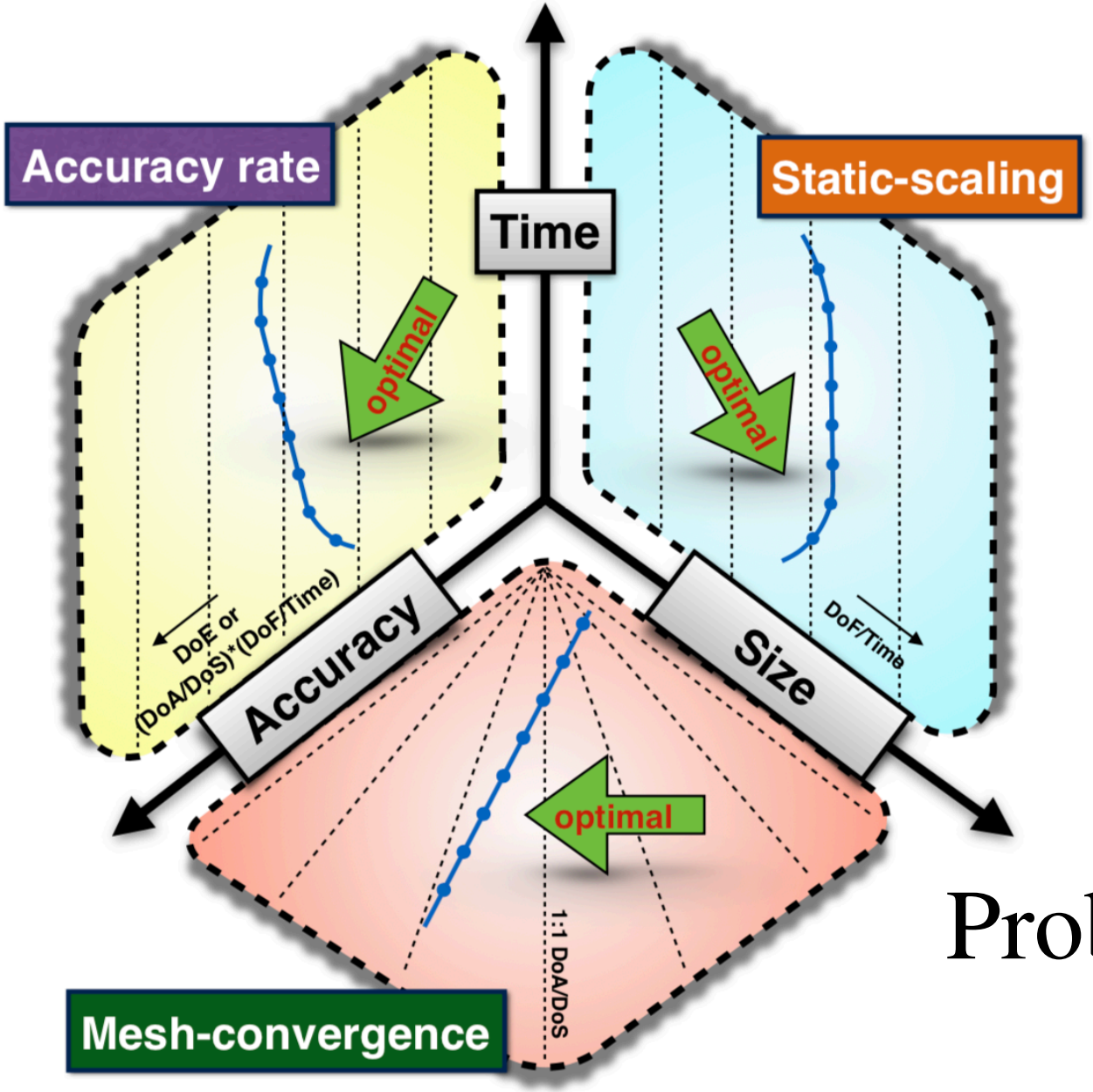
It distinguishes algorithms with different convergence behavior (Chang et al. 2018)


Accuracy rate
Static-scaling
Time
optimal
optimal
DoE or
(DoA/DoS)*(DoF/Time)
DoF/Time
Accuracy
Size
optimal
1:1 DoA/DoS
Mesh-convergence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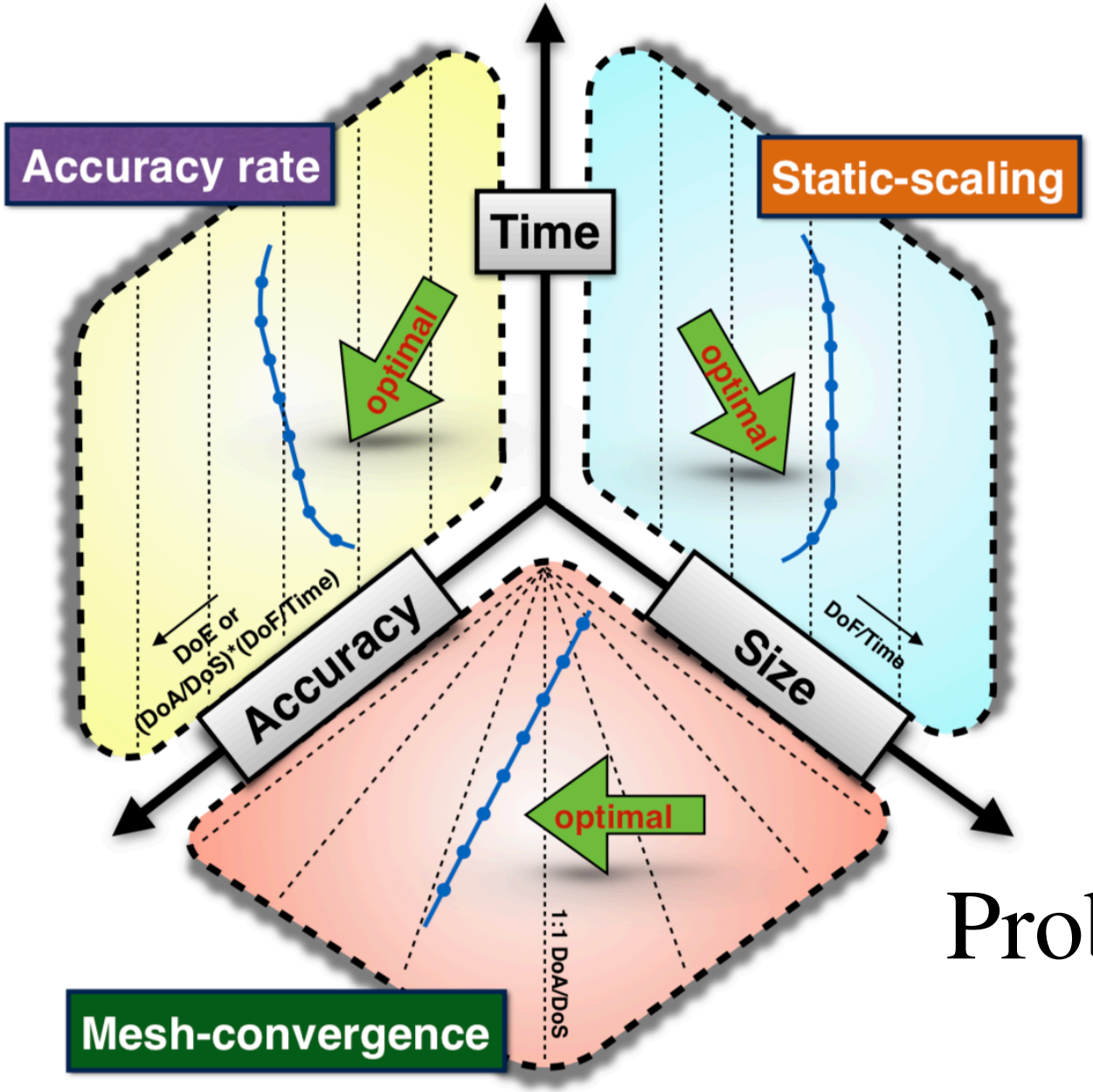
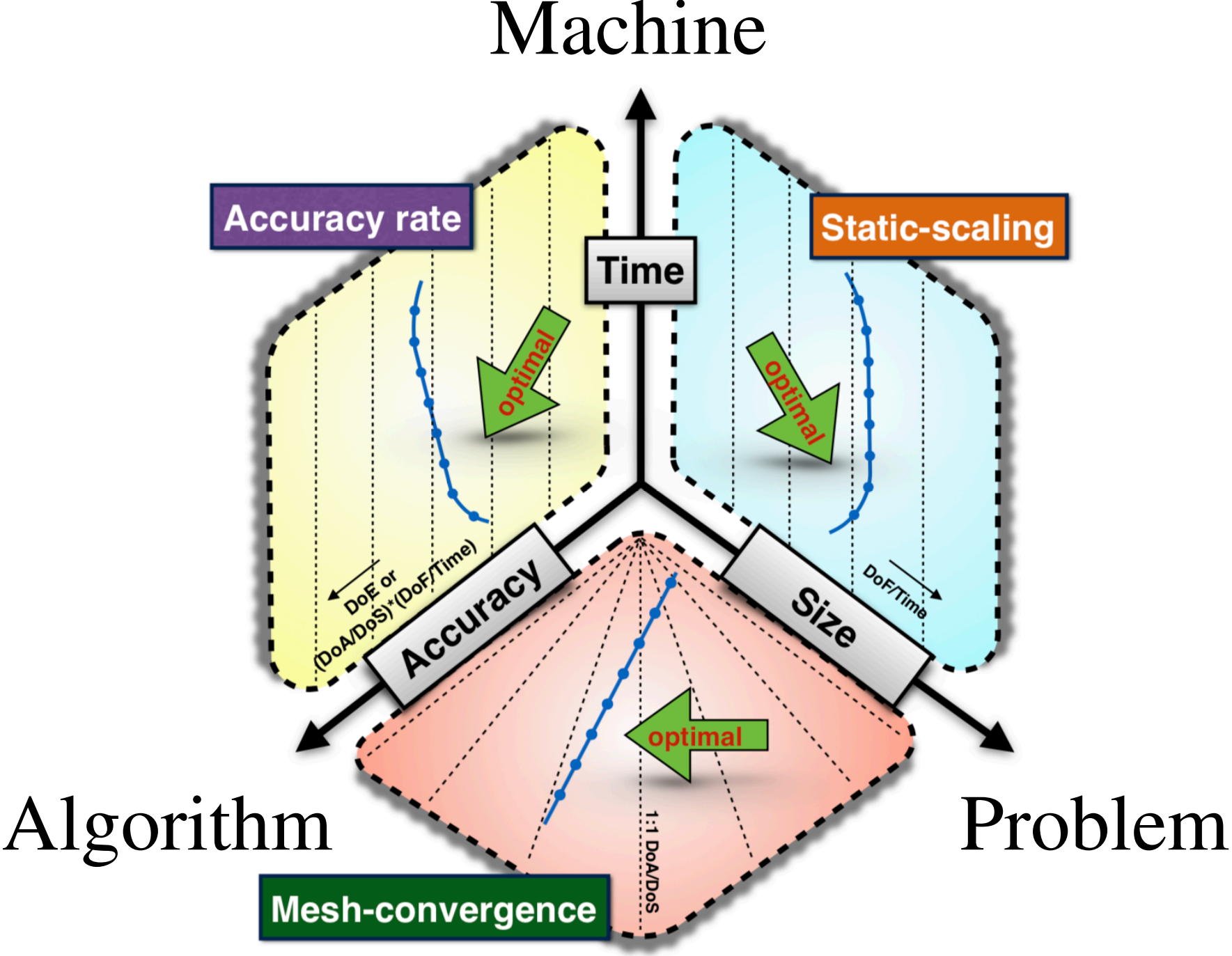
Problem

Machine



Problem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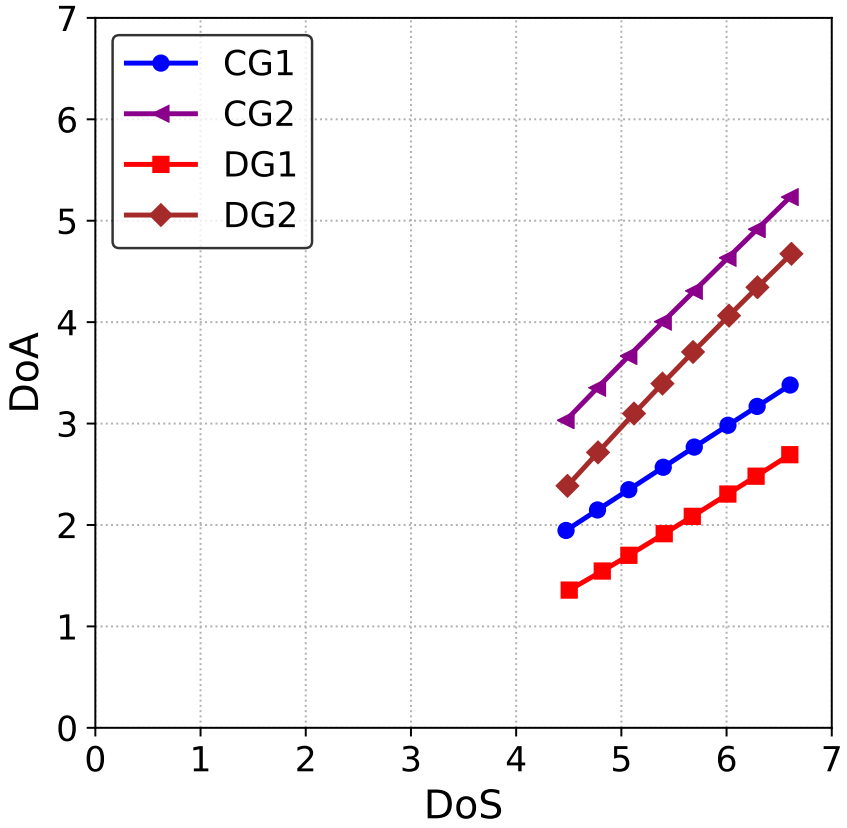
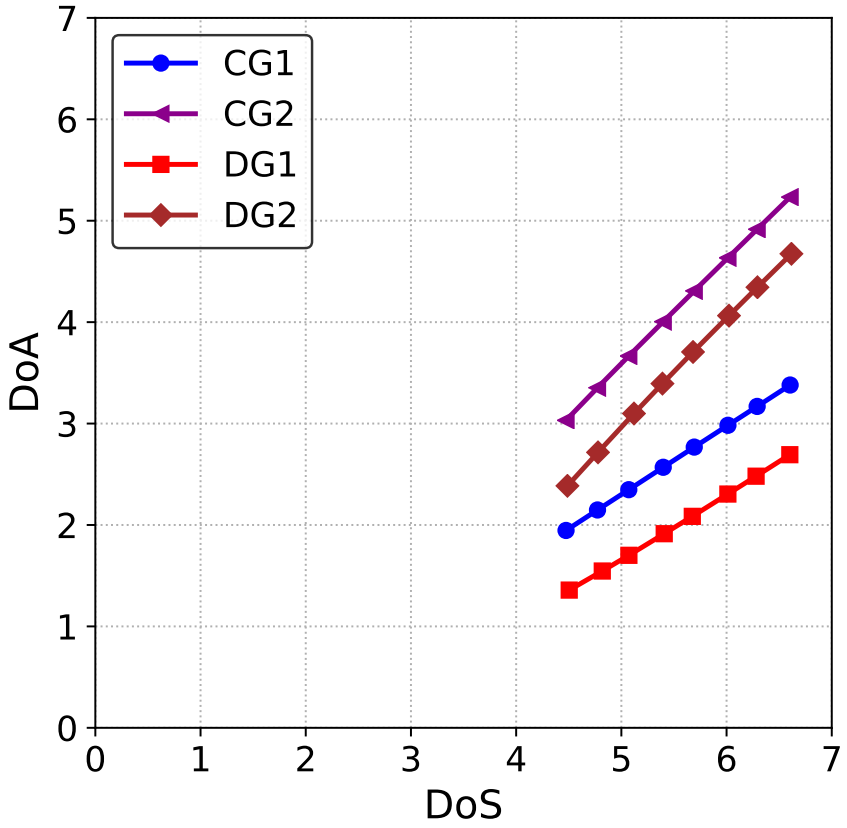
Algorithm
Problem
Mesh-convergence

Mesh Convergence Diagram

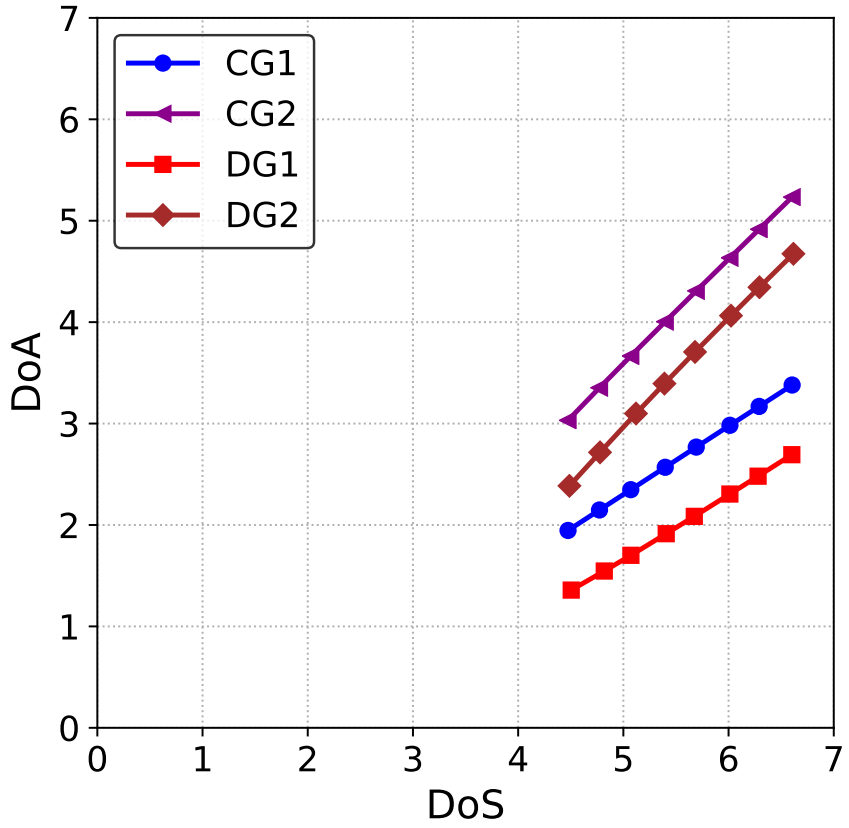


Mesh Convergence Diagram



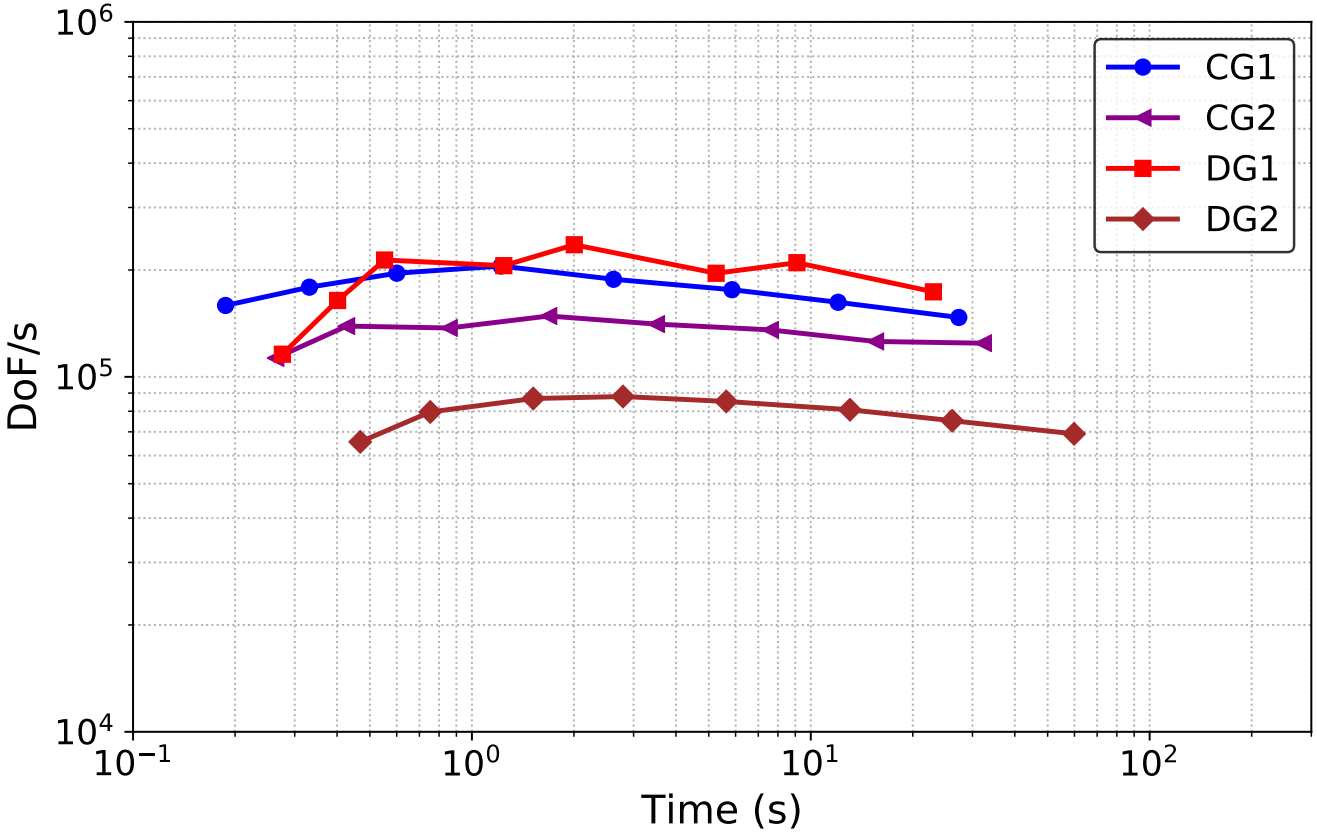
1/error vs. size

Mesh Convergence Diagram

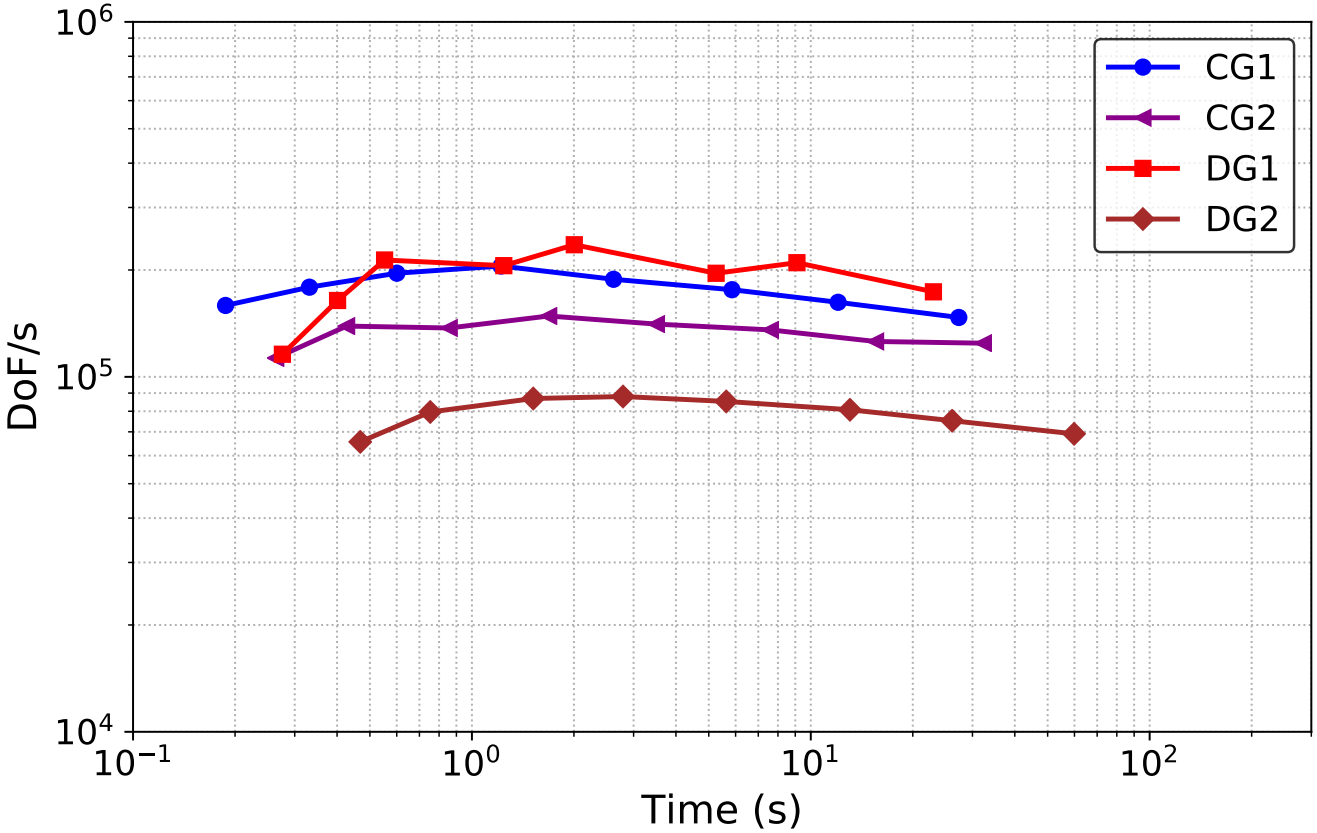


Does my Algorithm solve this Problem?

Static Scaling Diagram

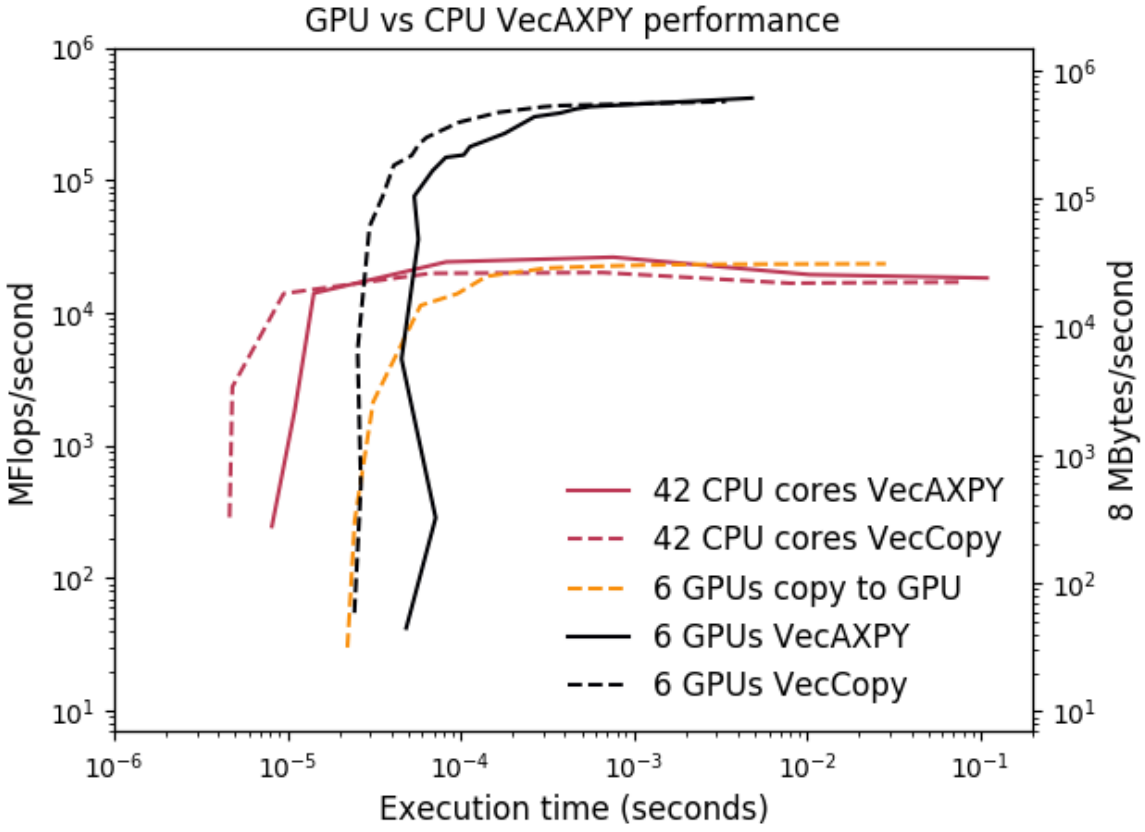


Static Scaling Diagram



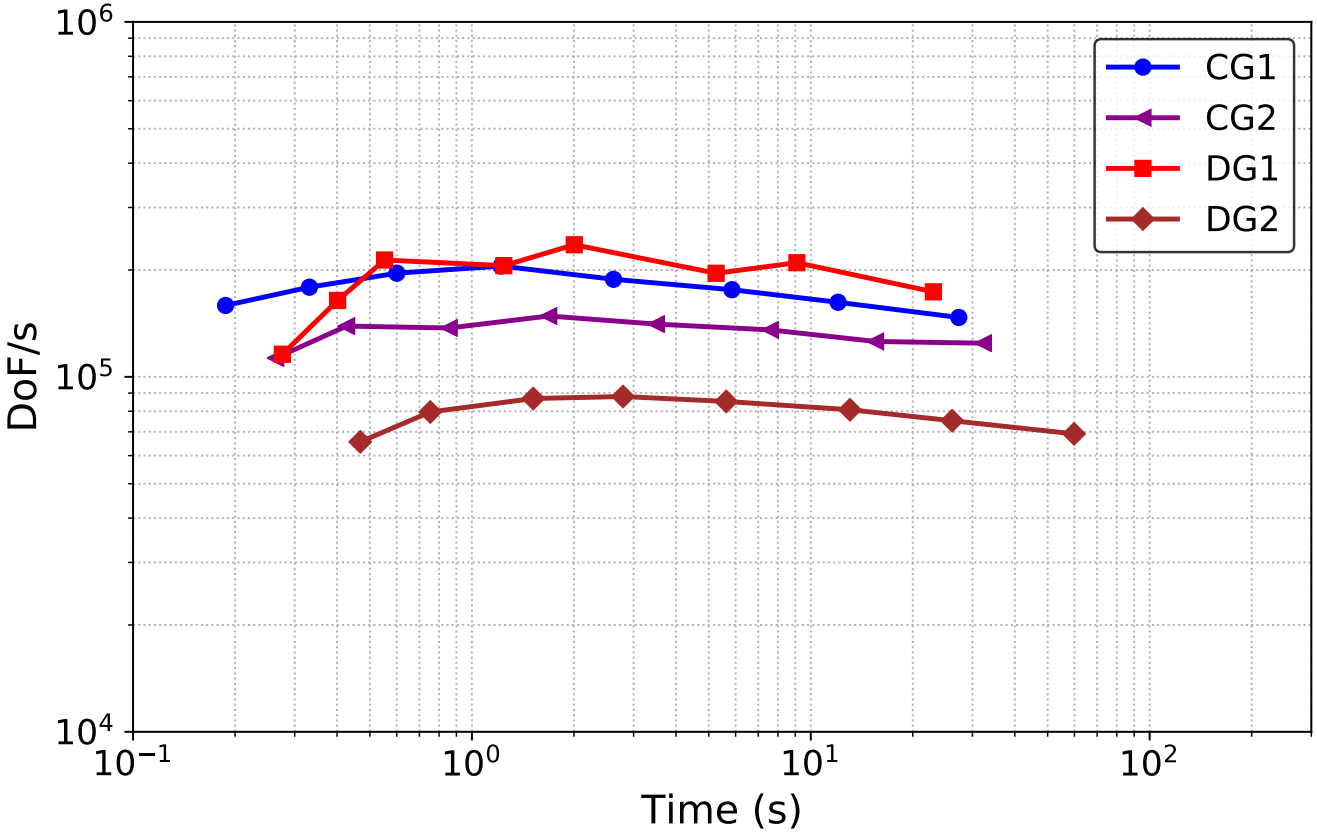
size/time vs. time

Static Scaling Diagram



size/time vs. time

Static Scaling Diagram



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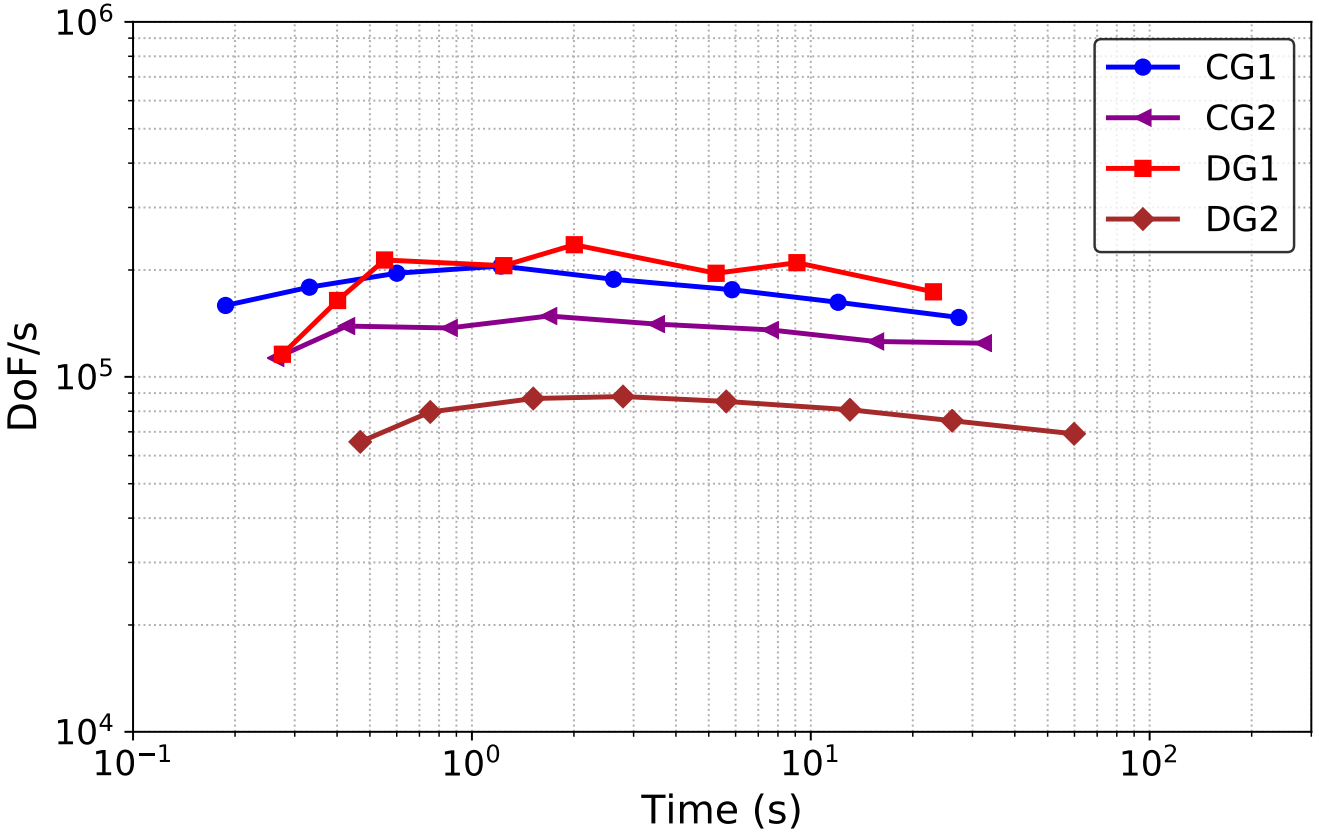
Static Scaling Diagram



Is my Algorithm efficient on this Machine?

How should we measure accuracy?

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accuracy rate $\frac{e}{T}$

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Marginal accuracy rate falls off steeply with problem size

Consider an optimal PDE solver:

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$$T = Wh^{-d} \text{ and } e = Ch^{\alpha}$$

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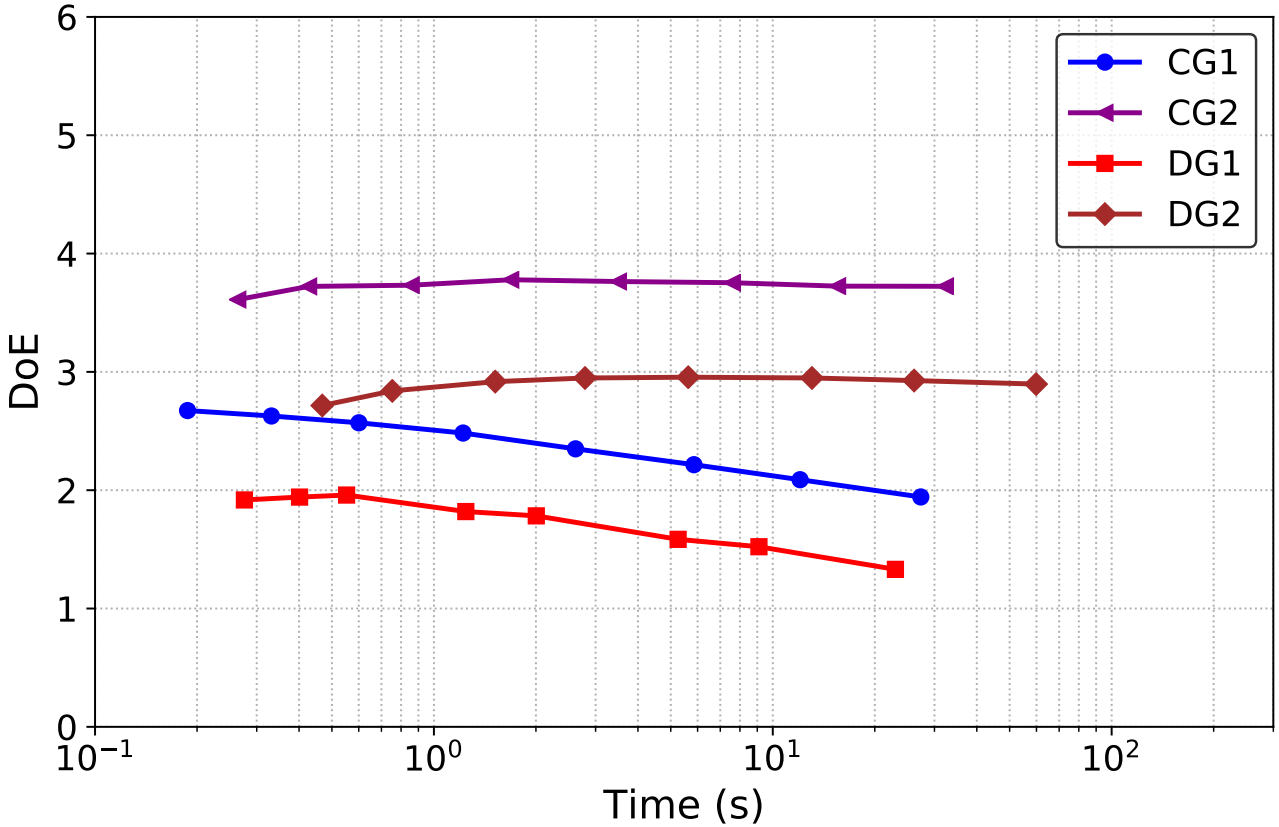
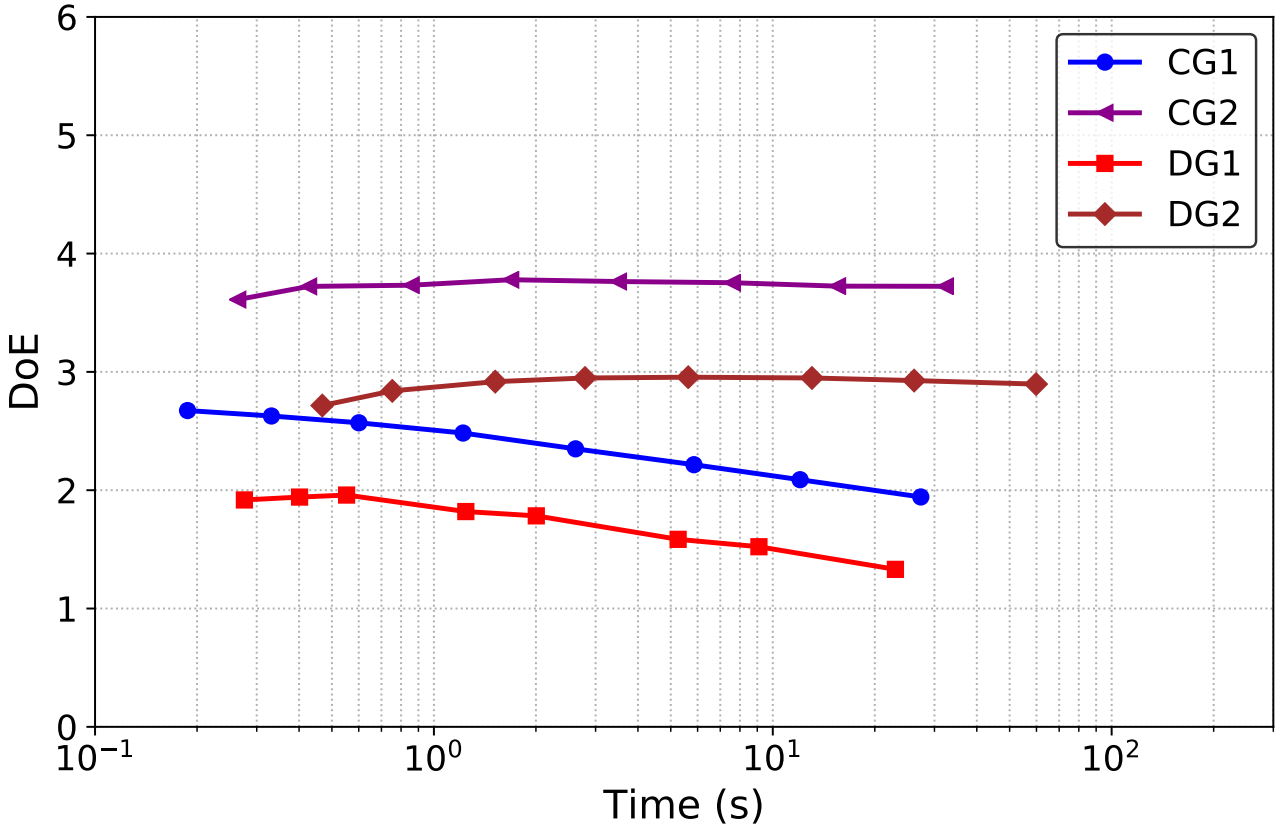
The error-time has a simple form

$$\begin{aligned} & -\log(e \cdot T) \\ = & -\log\left(Ch^{\alpha}Wh^{-d}\right) \\ = & (d-\alpha)\log(h) - \log(CW) \end{aligned}$$

Efficacy Diagram

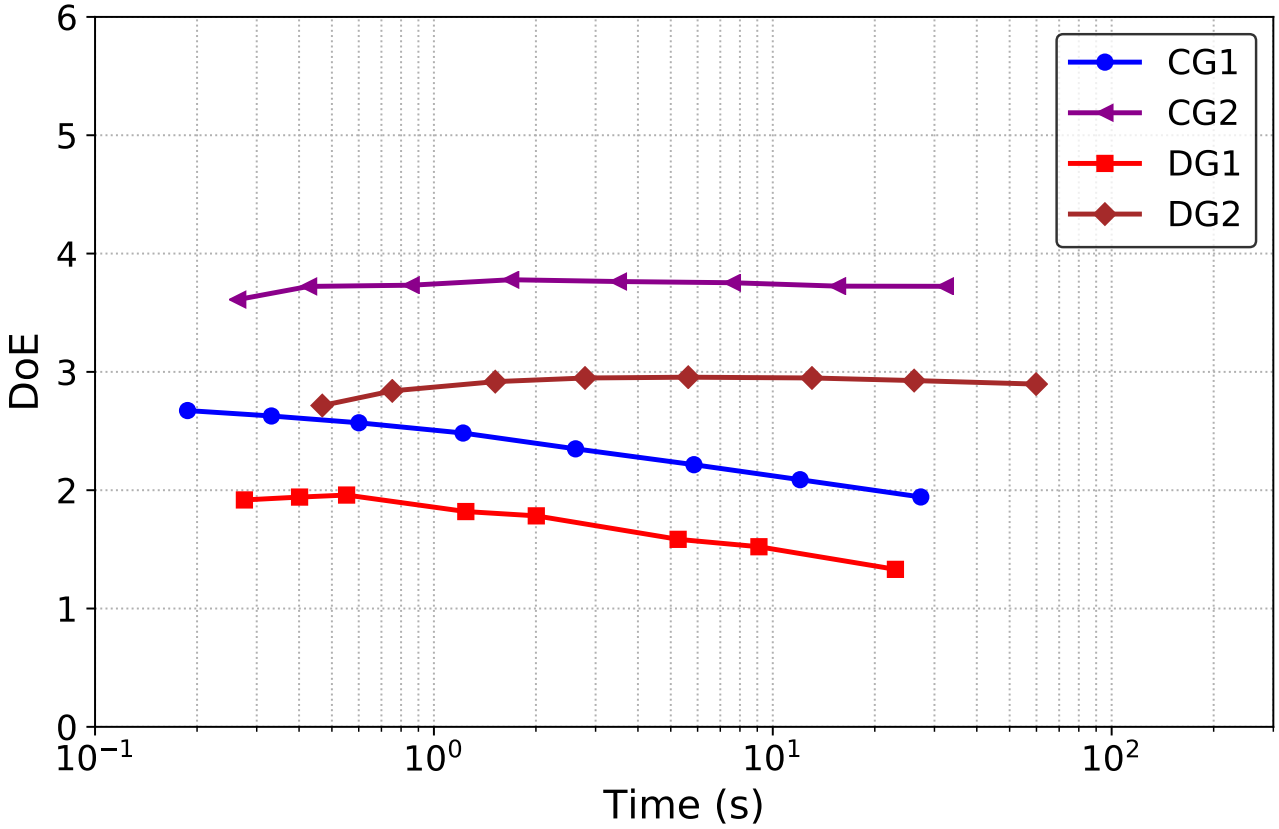


Efficacy Diagram



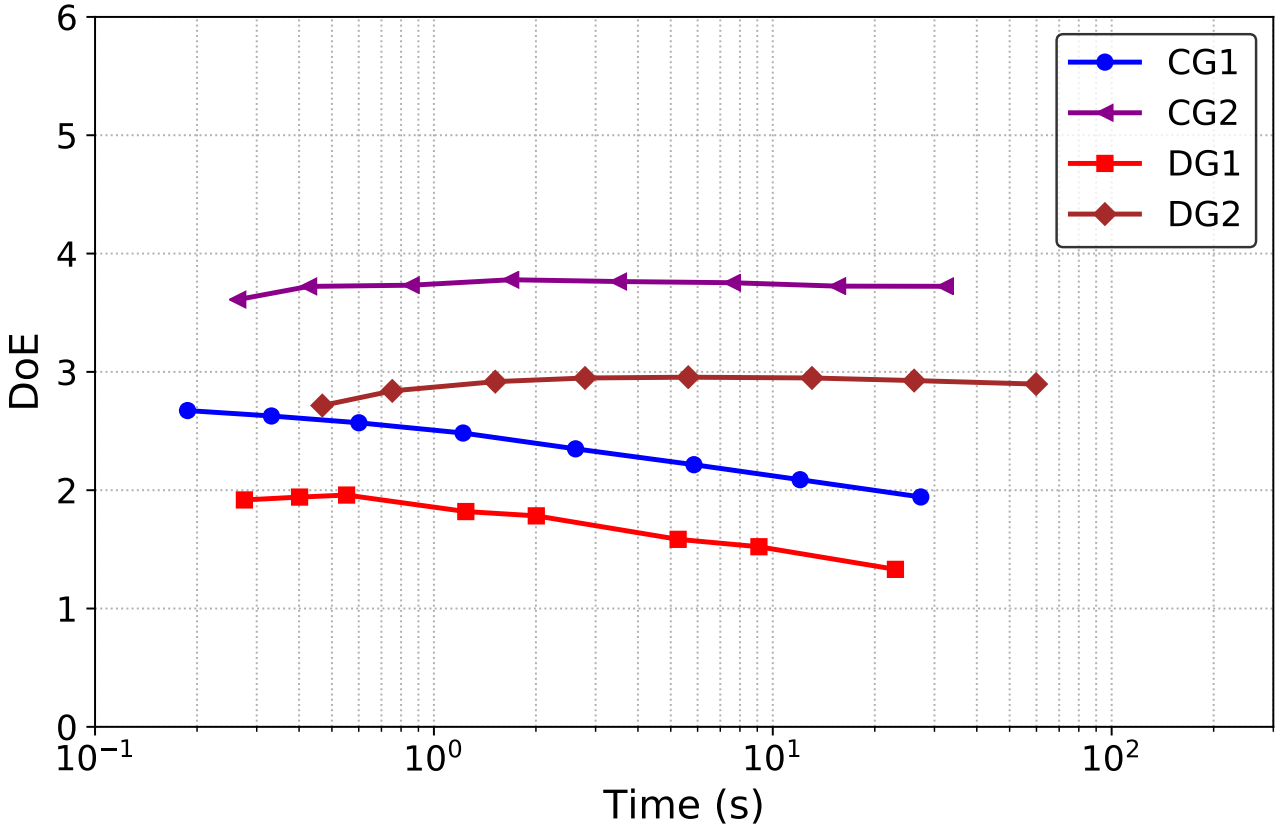
1/error-time vs. time

Efficacy Diagram



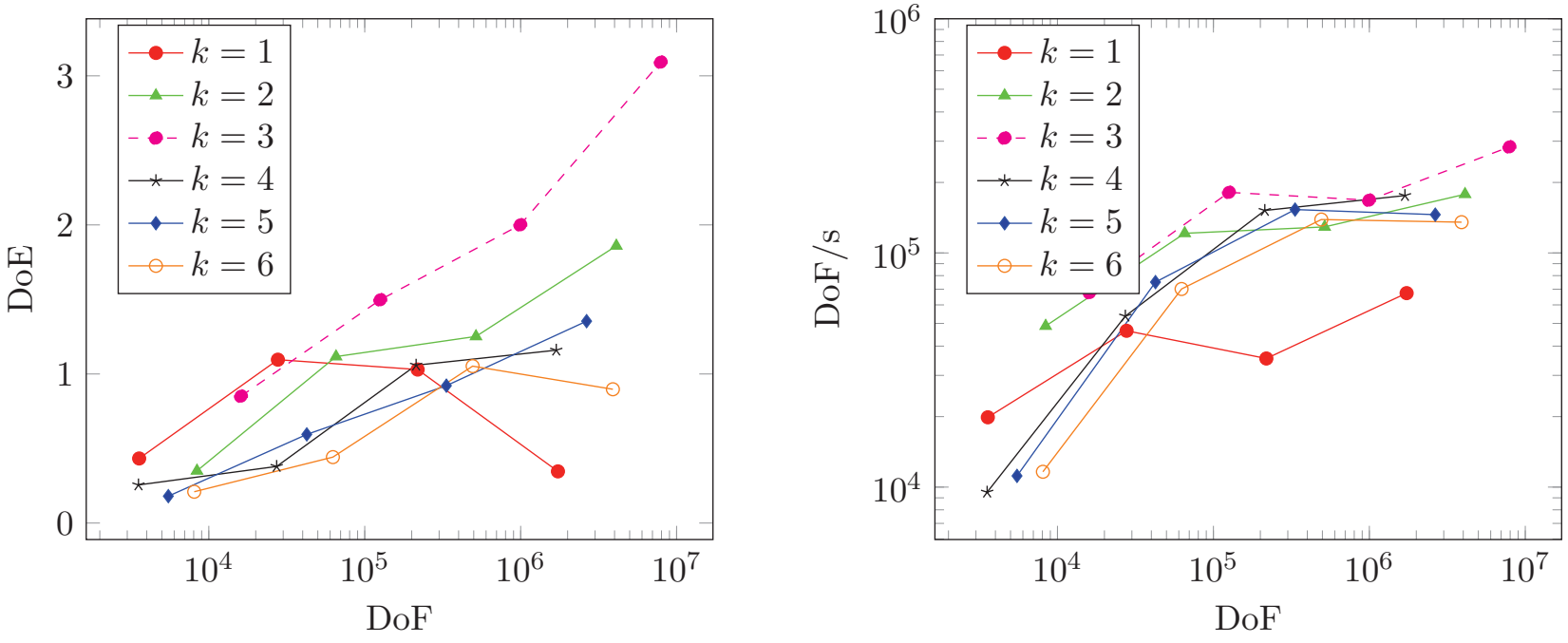
$$\frac{1/\text{error}}{\text{size}} \times \frac{\text{size}/\text{time}}{\text{time}} = \frac{1/(\text{error}\cdot\text{time})}{\text{time}}$$

Efficacy Diagram



Does my Algorithm solve this Problem efficiently on this Machine?

Efficacy vs. Static Scaling



(a) DoE vs DoF

(b) DoF/s vs DoF

Figure 17: Time-accuracy performance analysis for the nearly incompressible problem ($\lambda = 10^6$).

(Fabien 2019)

What else could we analyze?

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Communication-Avoiding (CA) algorithms have exciting lower bounds

(Ballard et al. 2011)

What else could we analyze?

CA TSQR is a great success

(Demmel et al. 2012)

What else could we analyze?

CA Krylov not a success

What else could we analyze?

CA Krylov not a success

Accuracy depends on coarse grid communication in preconditioner

Future Questions:

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Is there a variational characterization of optimal algorithms?

Future Questions:

Can we think of error-time as an Algorithmic Action?

References I

Chang, Justin, Maurice S. Fabien, Matthew G. Knepley, and Richard T. Mills (2018). "Comparative study of finite element methods using the Time-Accuracy-Size (TAS) spectrum analysis". In: SIAM Journal on Scientific Computing 40.6, pp. C779–C802. DOI: 10.1137/18M1172260. eprint: 1802.07832.

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Demmel, James, Laura Grigori, Mark Hoemmen, and Julien Langou (2012). "Communication-optimal Parallel and Sequential QR and LU Factorizations". In: SIAM Journal on Scientific Computing 34.1, A206–A239. DOI: 10.1137/080731992.