Geodynamic Simulator Building

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SIAM Parallel Processing, Tokyo, Kantō JP March 10, 2018





A Simulator is more Useful when the Researcher

Builds it Themselves

Outline

- Flexible Meshing
 - PyLith
 - DMNetwork
- Interaction of Discretizations and Solvers

How do I handle

many different mesh types
simply and efficiently?

PP18

Most packages handle one kind of mesh, or have completely separate code paths for different meshes

5/36

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6/36

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parallel distribution and load balancing, traversal for function/operator assembly, coarsening and refinement, generation of missing edges/faces, and surface extraction,

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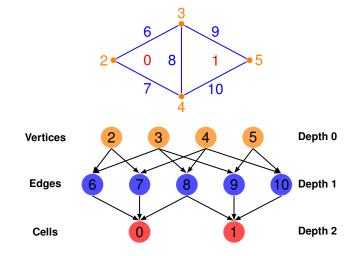
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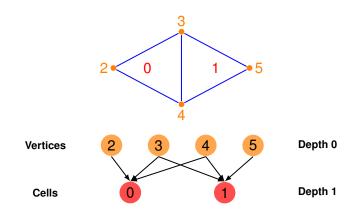
parallel distribution and load balancing, traversal for function/operator assembly, coarsening and refinement, generation of missing edges/faces, and surface extraction,

just **once**.

Interpolated triangular mesh

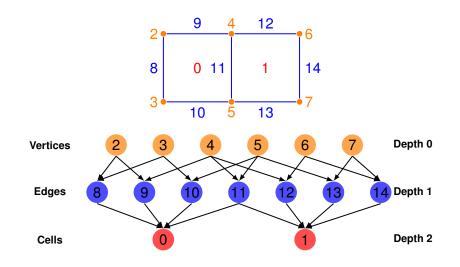


Sample Meshes Optimized triangular mesh

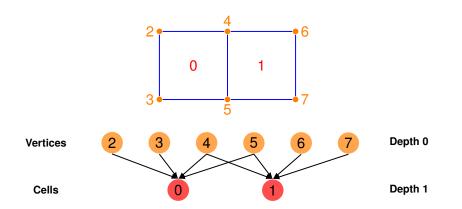


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Interpolated quadrilateral mesh

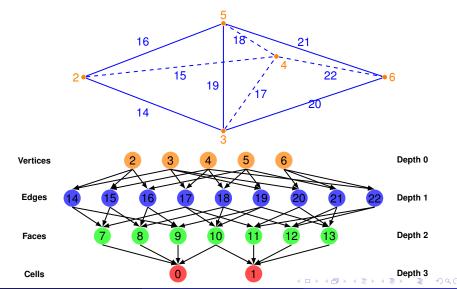


Optimized quadrilateral mesh



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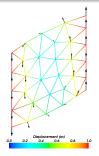
Interpolated tetrahedral mesh



Outline

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 - PyLith
 - DMNetwork





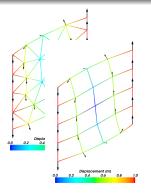
Many cell types

Surface extraction

Hybrid meshes

Aagaard, Knepley, Williams, J. of Geophysical Research, 2013.





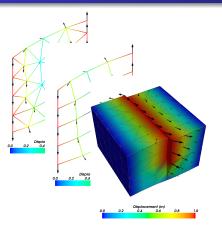
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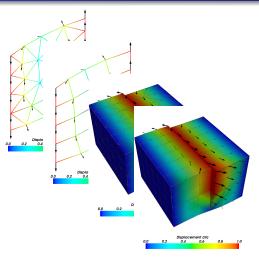
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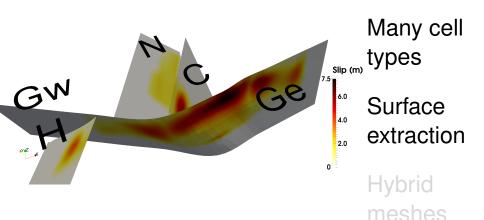


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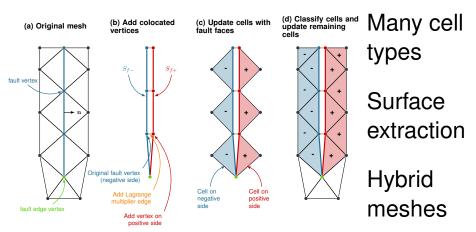
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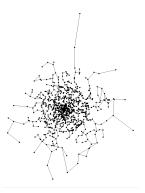


Example: DMNetwork

Plex on 30K cores of Edison for a finite volume hydraulic flow application.

Table IV. Execution Time of Transient State on Edison

No. of	Variables	Maximum Variables	Linear Preconditioner		
Cores		per Core	Block Jacobi	ASM	ASM
	(in millions)	(in thousands)		ov. 1	ov. 2
240	16	106	9.9 (48)	7.3 (25)	6.4 (20)
960	63	106	10.6 (55)	7.0(24)	6.2(20)
3,840	253	106	10.4 (53)	7.3(24)	6.7(20)
15,360	1,012	104	11.9 (53)	11.4(26)	9.9(20)
30,720	2,023	117	20.0 (53)	17.6 (26)	17.2 (20)



Maldonado, Abhyankar, Smith, Zhang, ACM TOMS, 2017

Outline

- Flexible Meshing
- Interaction of Discretizations and Solvers
 - PCTelescope
 - GMG with coefficients
 - Comparison of Discretizations

Main Question

How do I handle
many different discretizations
simply and efficiently?

Most packages handle one discretization,

FEniCS/Firedrake is a notable exception, and interface poorly with solvers, especially hierarchical solvers.

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The Section abstraction allows us to write code for

parallel data layout, block/field decompositions, (variable) point-block decompositions, removing Dirichlet conditions, (nonlinear) hierarchical rediscretization, and partial assembly (BDDC/FETI),

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PETSc Strategy

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```

just **once**.

A Section is a map

mesh point \Longrightarrow (size, offset)

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Data Layout mesh point \Longrightarrow # dofs

Boundary conditions mesh point ⇒ # constrained dofs

Fields mesh point \Longrightarrow # field dofs

A Section is a map

mesh point \Longrightarrow (size, offset)

Decouples Mesh, Discretization, and Solver

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Assembly gets dofs on each point and mesh traversal, no need for discretization spaces

A Section is a map

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Decouples Mesh, Discretization, and Solver

Solver gets data layout and ordering, no need for mesh traversal

A Section is a map

mesh point \Longrightarrow (size, offset)

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Solver gets field and point blocking, no need for discretization spaces

A Section is a map

mesh point \Longrightarrow (size, offset)

Decouples Mesh, Discretization, and Solver

Provides interface layer between PETSc and discretization packages Firedrake and LibMesh

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PCTelescope abstracts the parallel distribution of a linear system, so that

May, Sanan, Rupp, Knepley, Smith, PASC, 2016. slides



PCTelescope abstracts the parallel distribution of a linear system, so that

a user can bring their coarse level onto a single process for a direct solve,

```
-pc_type mg
-pc_mg_levels N
-mg_coarse_pc_type telescope
-mg_coarse_pc_telescope_reduction_factor nc
-mg_coarse_telescope_pc_type lu
```

May, Sanan, Rupp, Knepley, Smith, PASC, 2016. slides



PCTelescope abstracts the parallel distribution of a linear system, so that

or recreate the solver from the Gordon Bell Prize Winner 2015.

```
-pc_type mg
-pc_mg_levels NR
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-mg_coarse_pc_telescope_reduction_factor r
-mg_coarse_telescope_pc_type mg
-mg_coarse_telescope_pc_mg_levels NG
-mg_coarse_telescope_pc_mg_galerkin
-mg_coarse_telescope_mg_coarse_pc_type gamg
```

May, Sanan, Rupp, Knepley, Smith, PASC, 2016. slides



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The paper shows scaling up to 32³ processors on Piz Daint.

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The paper shows scaling up to 32³ processors on Piz Daint, and also hybrid CPU-GPU solvers.

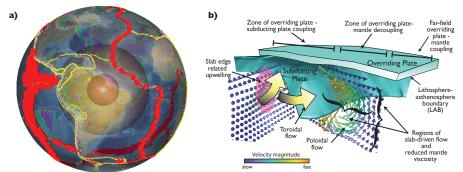
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Outline

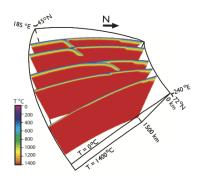
- Interaction of Discretizations and Solvers
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Regional mantle convection has highly variable viscosity, due to temperature and strain rate.



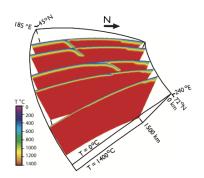
Jadamec, Billen, Nature, 2009.

We will specify an initial temperature, on some initial mesh, and let strain develop self-consistently.



Jadamec, Billen, Nature, 2009.

This temperature must be distributed, matching the mesh partition, and interpolate/restrict to meshes.



Jadamec, Billen, Nature, 2009.

Create Section mapping temperature to coarse cells, using PetscFECreateDefault() for a DG function space

Distribute the coarse mesh, using DMPlexDistribute()

Distribute the cell temperatures, using DMPlexDistributeField()

Transfer cell temperatures to finer cells (purely local)

Transfer cell temperatures to vertices on fine grid (purely local)

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Interpolation is straightforward

```
DMRefine(coarseMesh, comm, &fineMesh);
DMCreateInterpolation(coarseMesh, fineMesh, &I, &Rscale);
MatMult(I, coarseTemp, fineTemp);
```



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Now we restrict the input temperature to coarser meshes.

```
DMCreateInterpolation(coarseMesh, fineMesh, &I, &Rscale);
MatMultTranspose(I, fineTemp, coarseTemp);
VecPointwiseMult(coarseTemp, coarseTemp, Rscale);
```

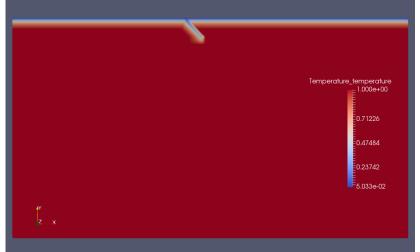


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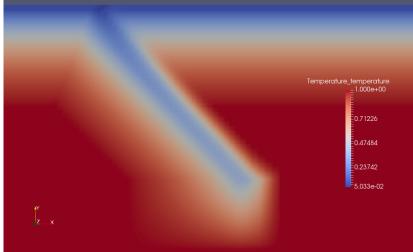


Mantle Temperature (Fine Grid, Level 3)

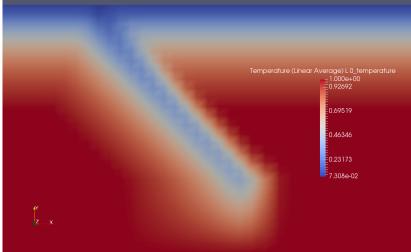


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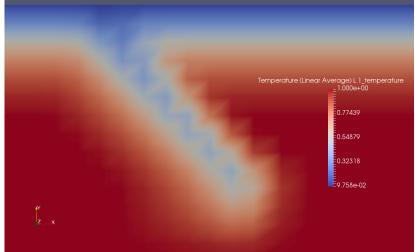
Mantle Temperature (Fine Grid, Level 3)



Mantle Temperature (Level 2, Q1 Restriction)



Mantle Temperature (Level 1, Q1 Restriction)



Mantle Temperature (Level 0, Q1 Restriction)



The power mean could better preserve low temperatures

$$\bar{x} = \left(\sum_{i} x_{i}^{p}\right)^{\frac{1}{p}}$$

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```
DMCreateInterpolation(coarseMesh, fineMesh, &I, &Rscale);
MatShellSetOperation(I, MATOP_MULT_TRANSPOSE,
    MatMultTransposePowerMean_SeqAlJ);
MatMultTranspose(I, fineTemp, coarseTemp);
VecPointwiseMult(coraseTemp, coarseTemp, Rscale);
VecPow(coarseTemp, p);
```

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The power mean could better preserve low temperatures

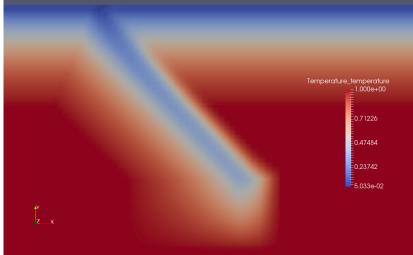
$$\bar{x} = \left(\sum_{i} x_{i}^{p}\right)^{\frac{1}{p}}$$

```
for (i = 0; i < m; ++i) {
  idx = a->j + a->i[i];
  v = a->a + a->i[i];
  n = a->i[i+1] - a->i[i];
  xi = x[i];
  for (j = 0; j < n; ++j) {
    y[idx[j]] += v[j]*PetscPowScalarReal(xi, p);
  }
}</pre>
```

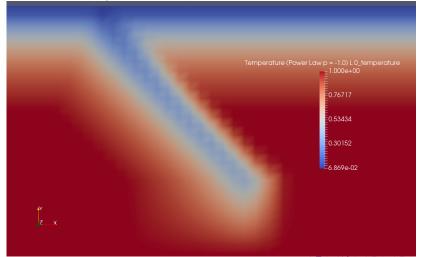
It reuses the parallel MatMultTranspose() implementation.

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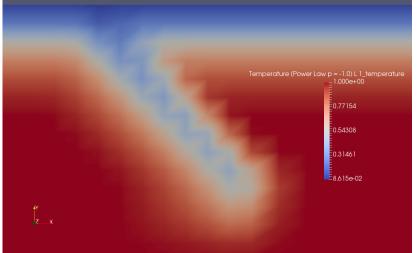
Mantle Temperature (Fine Grid, Level 3)



Mantle Temperature (Level 2, Harmonic Restriction)



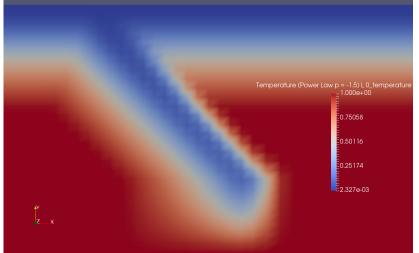
Mantle Temperature (Level 1, Harmonic Restriction)



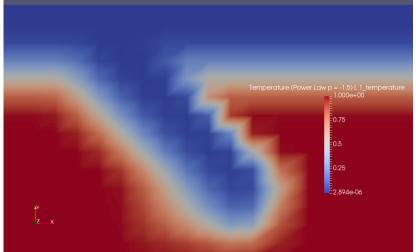
Mantle Temperature (Level 0, Harmonic Restriction)



Mantle Temperature (Level 2, p = -1.5 Restriction)



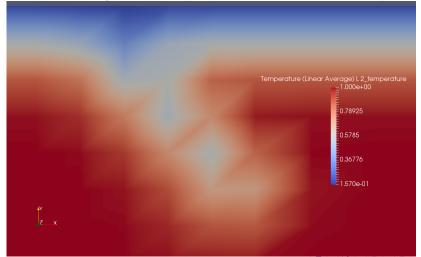
Mantle Temperature (Level 1, p = -1.5 Restriction)



Mantle Temperature (Level 0, p = -1.5 Restriction)



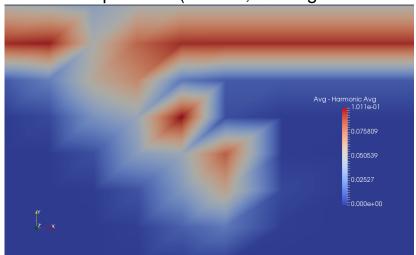
Mantle Temperature (Level 0, Q1 Restriction)



Mantle Temperature (Level 0, Harmonic Restriction)



Mantle Temperature (Level 0, Q1 Avg - Harmonic Avg)



```
-snes_rtol 1e-7 -snes_atol 1e-12 -snes_linesearch_maxstep 1e20
  -ksp rtol 1e-5
  -pc type fieldsplit
    -pc_fieldsplit_diag_use_amat
    -pc fieldsplit type schur
    -pc_fieldsplit_schur_factorization_type full
    -pc_fieldsplit_schur_precondition all
      -fieldsplit_velocity_ksp_type gmres
        -fieldsplit_velocity_ksp_rtol 1e-8
      -fieldsplit_velocity_pc_type mg
        -fieldsplit_velocity_pc_mg_levels n
          -fieldsplit_velocity_mq_levels_ksp_type gmres
            -fieldsplit velocity mg levels ksp max it 4
            -fieldsplit_velocity_mq_levels_pc_type pbjacobi
            -fieldsplit_velocity_mg_levels_pc_pbjacobi_variable
            -fieldsplit_velocity_mg_levels_pc_use_amat
      -fieldsplit_pressure_pc_type asm
        -fieldsplit pressure sub pc type ilu
        -fieldsplit_pressure_ksp_rtol 1e-4
        -fieldsplit pressure ksp max it 20
```

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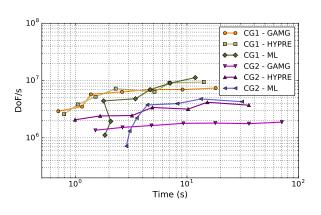
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Outline

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Example: TAS Performance Analysis

Static Scaling (1K procs)



Measure dof/s,

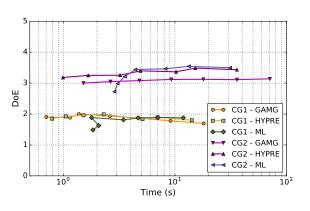
fixed parallelism,

different sizes.

Chang, Fabien, Knepley, Mills, submitted, 2018.

Example: TAS Performance Analysis

Accuracy Scaling (1K procs)



Measure error × time

fixed parallelism

different sizes.

Chang, Fabien, Knepley, Mills, submitted, 2018.

Conclusions

Abstractions for Topology and Geometry, Data Layout, and **Operator Composition** let the User construct the Simulator.

Conclusions

http://bitbucket.org/petsc/petsc http://github.com/petsc/petsc



Example: Magma Dynamics

Show magma performance using FAS

Knepley, Melt in the Mantle Program, Newton Institute, 2016.