Computational Complexity / Decision Making (at Chess)

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Computational Complexity

- The study of the time *needed* to solve computational problems, and how much memory and other resources computers require.
- Largely independent of the computer model, beyond a fundamental divide into **serial**, **parallel**, and **quantum**.
- Main technical achievement: the relation of computational problems by **reducibility**.
- Main scientific surprise:

> The **many thousands** of computational problems that have been studied in many disciplines, some for centuries, cluster into **barely over a dozen** equivalence classes under reducibility.

- The biggest cluster is the class of **NP-complete** problems.

P=NP and Worse

- **P**: problems with algorithms that **solve** them in **polynomial time**:

> As the size of the data doubles, the time needed goes up by at most a **linear** factor: $t(n) = n^k \implies t(2n) \leq Kt(n)$, $K = 2^k$.

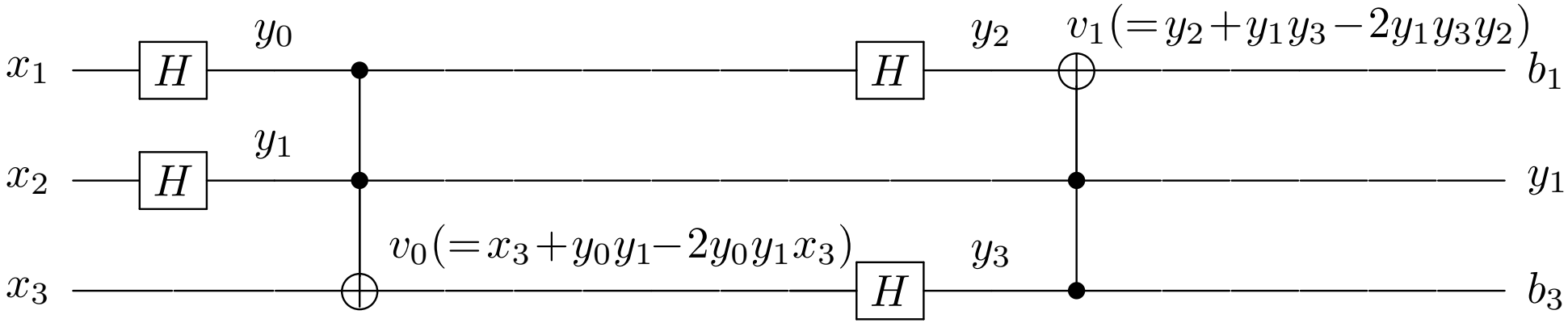
- **NP**: "Nondeterministic" Polynomial Time: If you know a secret fact or guess a good answer, you can verify and **teach** it to someone in polynomial time.
- Example: Given a Boolean formula f like

$$f = (x_1 \vee (\neg x_2)) \wedge ((\neg x_1) \vee x_2 \vee x_3) \wedge ((\neg x_2) \vee (\neg x_3)),$$

  is there a way to make f true?
- Called *Satisfiability* (SAT).
- Equivalent to $\neg f$ *not* being a **tautology**.
- Is NP-complete, so NP = P $\iff$ SAT belongs to P.
- We don't even know whether SAT can be solved in **linear** time!

Other Problems and Models

- **Factoring** is among a handful of problems in NP not known to be complete or in P.
- RSA security depends on it, so many want it to be *hard*.
- But solvable in polynomial time by a **quantum computer**.
- Textbook on quantum algorithms; blog series: Can QCs be Built?
- Research on simulating **quantum circuits** by logic and algebra:

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p1 = p0 XOR (y0 & x1)   p2 = p1 XOR (y1 & x2)   v0 = x3 XOR (y0 & y1)
p3 = p2 XOR (y2 & y0)   p4 = p3 XOR (y3 & v0)   v1 = y2 XOR (y3 & y1)
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Decision Making in Chess... and Tests

The ____ of drug-resistant strains of bacteria and viruses has ____ researchers' hopes that permanent victories against many diseases have been achieved.

(a) vigor . . corroborated

(b) feebleness . . dashed

(c) proliferation . . blighted

(d) destruction . . disputed

(e) disappearance . . frustrated

(source: itunes.apple.com)

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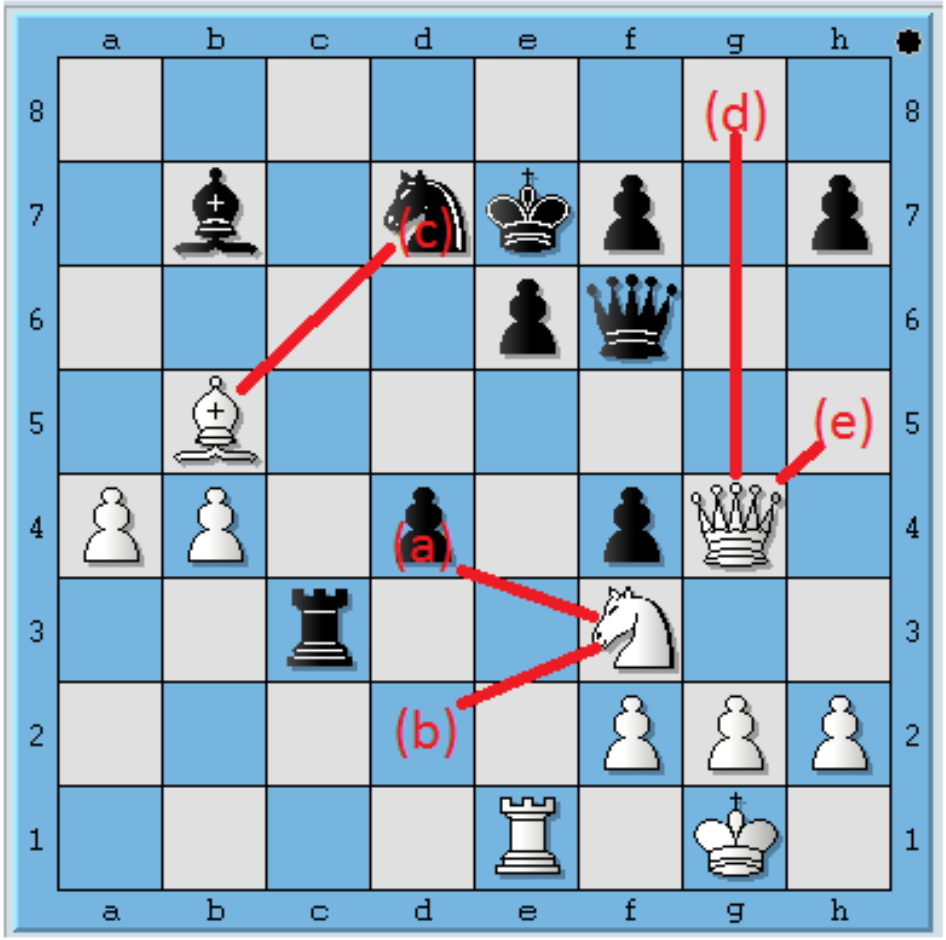

Advantages of Chess Model

1. **Large data**: tens of millions of moves in the public record of games.
2. **Known and Stable Standards**: Quality in chess measured by **Elo rating scale**.
3. **Depth** and **level** of thinking natural from structure of game.
4. **Intrinsic** formulation of **difficulty**.
5. **Tight correspondence** to **item-response theory** and other *psychometric* and decision-making models.
6. **Predictive Analytics**: can do risk evaluation, fraud detection...
7. Within chess: **intrinsic ratings** and **cheating testing**.
8. **Discover new scientific regularities of human thought processes.**