Name and St.ID#:____

CSE396, Spr'17 — Second Prelim Exam — Apr. 27, 2017

Closed books and laptops, one notes sheet allowed, closed neighbors, 75 minutes. Do ALL FOUR problems **on these exam sheets**. Extra sheet(s) may be requested later. Please *show all your work*—this may help for partial credit. The exam totals 100 pts., subdivided as shown. The alphabet Σ for all problems is $\{a, b\}$. The notation has all been seen before.

(1) 9 + 6 + 3 = 18 pts.

Consider the context-free grammar $G = (V, \Sigma, R, S)$ with $\Sigma = \{a, b\}$ and rules

$$\begin{aligned} S &\rightarrow TU \\ T &\rightarrow aTa \mid bTb \mid U \\ U &\rightarrow aU \mid bU \mid \epsilon \end{aligned}$$

(a) Create a grammar G' such that $L(G') = L(G) \setminus \{\epsilon\}$ and G' has no ϵ-rules. Along the way, say which variables of G are nullable.

(b) Show that G is ambiguous by giving two different parse trees for some string $x \in L(G)$.

(c) Is G comprehensive? "Comprehensive for what language?", you may ask. Does it matter?

(2) (12 + 12 + 6 = 30 pts. total)

Recall the definition of $\#u(x)$ as the number of times the substring u occurs within the string x, counting overlaps—so that for instance $\#aa(baaab) = 2$. Define $T = \{x : \#aa(x) = \#bb(x)\}$. Let $G = (V, \Sigma, R, S)$ be the context-free grammar with variables $V = \{S, A, B\}$ and rules

$$\begin{aligned} S &\rightarrow BSA \mid \epsilon \\ A &\rightarrow Aab \mid aab \\ B &\rightarrow baB \mid bba \end{aligned}$$

(a) Consider these four "target properties": (i) "begins with a," (ii) "ends with a," (iii) "begins with b," and (iv) "ends with b." Say which one(s) hold for strings derived from A, and which one(s) hold for strings derived from B.

(b) Use your targets in (a), and other facts you may add to them, to show that $L(G) \subseteq T$.

(c) Suppose G' adds the rule $S \rightarrow ASB$ to G. Is G' still sound, i.e., is $L(G') \subseteq T$? If you say yes, prove it; if you say no, give a leftmost derivation of a string $x \in L(G') \setminus T$.

(3) (18 + 15 + 4 = 37 pts.)

Define $L = \{ x \in \{ a, b \}^* : x = x^R \ \wedge \ \#a(x) = \#b(x) \}$.

(a) Prove using the CFL Pumping Lemma that L is not a context-free language. (Hint: Consider strings in L that also belong to $a^*(bb)^*a^*$.)

(b) Sketch in prose a 2-tape deterministic Turing machine M such that $L(M) = L$. Describe the operation of M as a finite sequence of "passes" and say what happens in each pass. Is your M a pushdown automaton? Could it be one?

(4) (15 pts.)

Define $r = b(aa)^*b$. Consider $S_0 = \{\epsilon, a, b, ba\}$. This is a PD set for $L(r)$, but it is not maximal. Find another string u such that $S = S_0 \cup \{u\}$ is a PD set of size 5 for $L(r)$. Show your work to verify this; drawing a DFA is optional and might be beside the point.

END OF EXAM