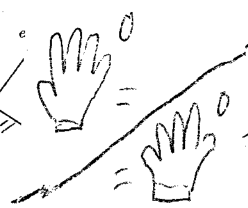
So this exemplifies what last Wednesday's (Jan 13) lecture said about $\emptyset$ being analogous to zero, and $\{\varepsilon\}$ behaving like 1.
But now we want to push the analogy to obey this law of exponents:

<u>Numbers</u>:	$a^i \cdot a^j = a^{i+j}$	Works when $i=0$ <u>if</u> $a^0 = 1$
<u>Strings</u>:	$x^i \cdot x^j = x^{i+j}$	OK for $i=0$ if $x^0 = \varepsilon$
<u>Languages</u>:	$A^i \cdot A^j = A^{i+j}$	ok for $i=0$ if $A^0 = \{\varepsilon\}$.

So we make a <u>convention</u>: For every language A, $A^0 = \{\varepsilon\}$.
And we declare this true even for $A = \emptyset$: $\emptyset^0 = \{\varepsilon\}$. <u>But why??</u>
How can we get something out of <u>nothing</u>? (lecture said $\{\varepsilon\}$ is <u>something</u>)

[Famous Buddhist "Koan" riddle: "What is the sound of one hand clapping?" I don't have an answer. But for <u>zero</u> hands I do: The sound of 0 hands clapping is the empty sound, ε. Then $\emptyset^0 = \{\varepsilon\}$ is "merely" the further step of saying this is true <u>even when there are no hands</u>.]



We would feel more comfortable about this if in math, <u>$0^0 = 1$</u>.
Here's the argument: In Discrete Math, for any sets P and Q, <u>Q^P</u> stands for the set of functions $f: P \to Q$. And by rule, $|Q^P| = |Q|^{|P|}$.
For example, let $P = \{0,1,2,3,4\}$ and $Q = \{0,1\}$. Then a function $f: P \to Q$ is the same as a binary string of length 5. E.g. the string 00110 <u>is</u> the function $f(0)=0$, $f(1)=0$, $f(2)=1$, $f(3)=1$, $f(4)=0$. There are $2^5 = 32$ such binary strings, and $|Q^P| = |Q|^{|P|} = 2^5$ as needed.
Hence, $\emptyset^\emptyset$ stands for the set of functions $f: \emptyset \to \emptyset$. Are there any?
Yes! $\emptyset$ is a function from $\emptyset$ to $\emptyset$. It's the only one, so $0^0 = |\emptyset|^{|\emptyset|} = |\emptyset^\emptyset| = 1$!