Topic: The Mathematics of Strings

Q: When is a decimal integer a multiple of 3?

Fact: Can add the digits. The sum is a multiple of 3 if and only if the original number is.

\[ x = 1056247 \]

Add digits: 25

Add digits: 7

Base case: Not 0, 3, 6, or 9, so answer is no.

1,000,000

Add digits: 2

1,099999

46

10

109998

Make \( x \) bigger did not make the digit sum bigger.

So "philosophically" the digit sum is not like in calculus:

Tiny changes to the number can affect the digit sum majority in either direction.

Vote that technically the algorithm involves recursion, with single digit as base cases.

\[
\begin{array}{c}
11011 \\
\times 11 \\
\hline
11011 \quad \text{base 2} \\
11011 \\
\downarrow \\
1010001 \\
\hline
= 81 \text{ base 10}
\end{array}
\]
2: How to write concatenation symbolically?

Let \( X = 101 \) and \( Y = 110 \)

Consider \( Z = 101110 \). Should we write

\[
Z = X + Y \quad \text{or} \quad Z = X \cdot Y?
\]

\[
\text{NB: } Z = X + Y \text{ intends } \#	ext{ as an actual char.}
\]

\[
W_2 = 101101 = XX
\]

\[
x^2 = W_2
\]

\[
x^3 = W_3 = 101101101 = X + X + X \quad \text{or} \quad X \cdot X \cdot X
\]

\[
x^4 = W_4 = 101101101101 = X + X + X + X \quad \text{or} \quad X \cdot X \cdot X \cdot X
\]

Consider \( Z' = 110101 \neq Z = 101110 \)

Write \( Z' = Y + X^2 \) or \( Z' = Y \cdot X^2 \)

\[
x^2 = W_2
\]

\[
x^3 = W_3 = X = 101
\]

\[
x^0 = \varepsilon \text{ def}
\]

\[
W_0 = '' \text{ empty something?}
\]

\[
\emptyset: \text{ empty set}
\]

\[
\varepsilon: \text{ empty string}
\]

\[
\text{Greek epsilon}
\]

\[
\text{Power Rule for Strings: } x^a \cdot x^b = x^{a+b}
\]

\( \lambda \): lambda also used.

Hence desirable to think of string concatenation as a kind of multiplication

Thus next week: bump up to sets of strings, called languages.