Tue Jan 27

Sets and Logic

[Continue from 3rd slide of overhead notes]

1

$197 + 48 = 245$

$197 + 48 + 48 = 293$

$197 + 48 + 48 + 48 = 341$

Introducing ---

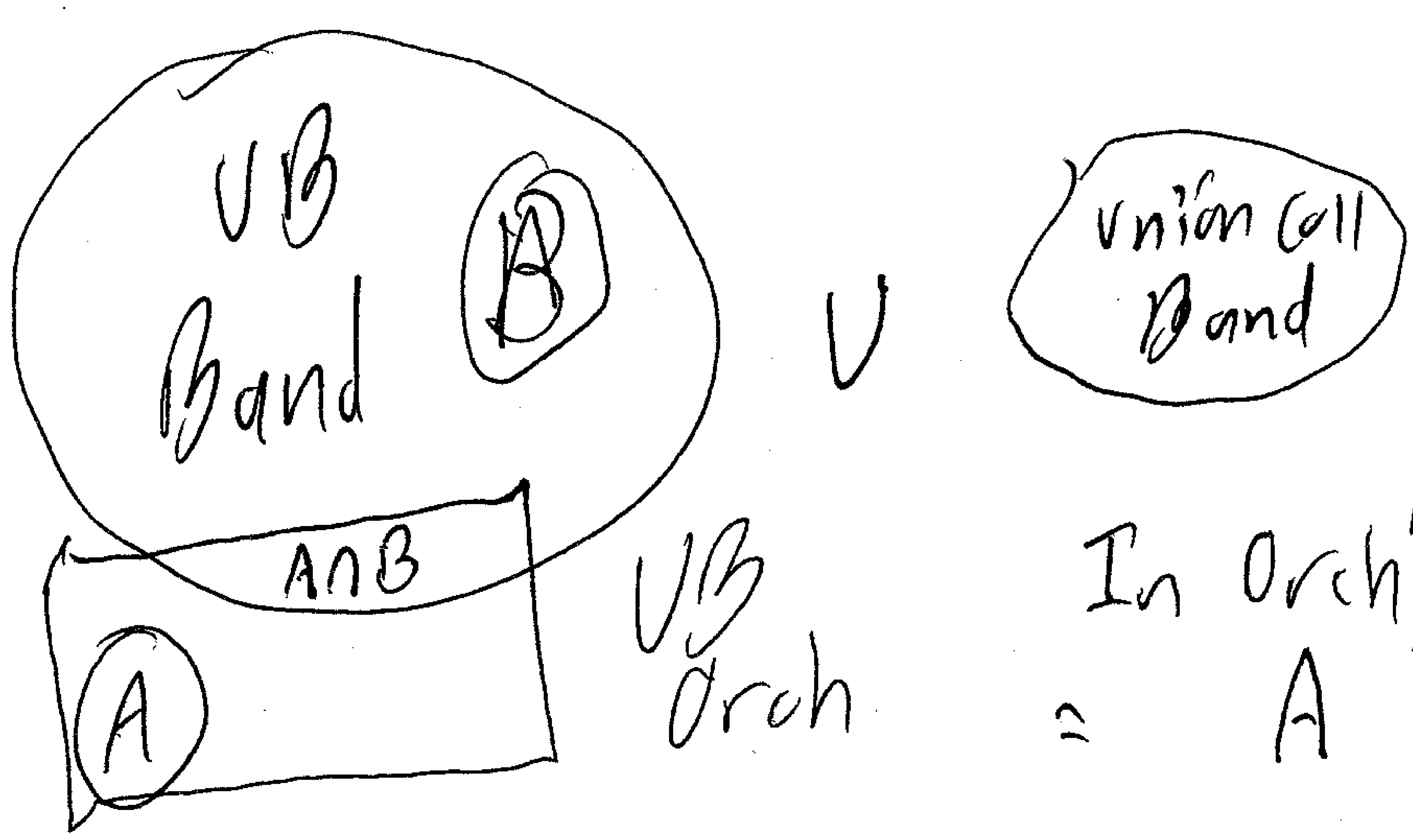
The UB Band + The Union College Band

OR

Person X

$X \in$ UB_Band $\vee$ $X \in$ Union_Band

$X \in$ UB $\vee$ $X \in$ Union $\vee$ $X \in$ Union



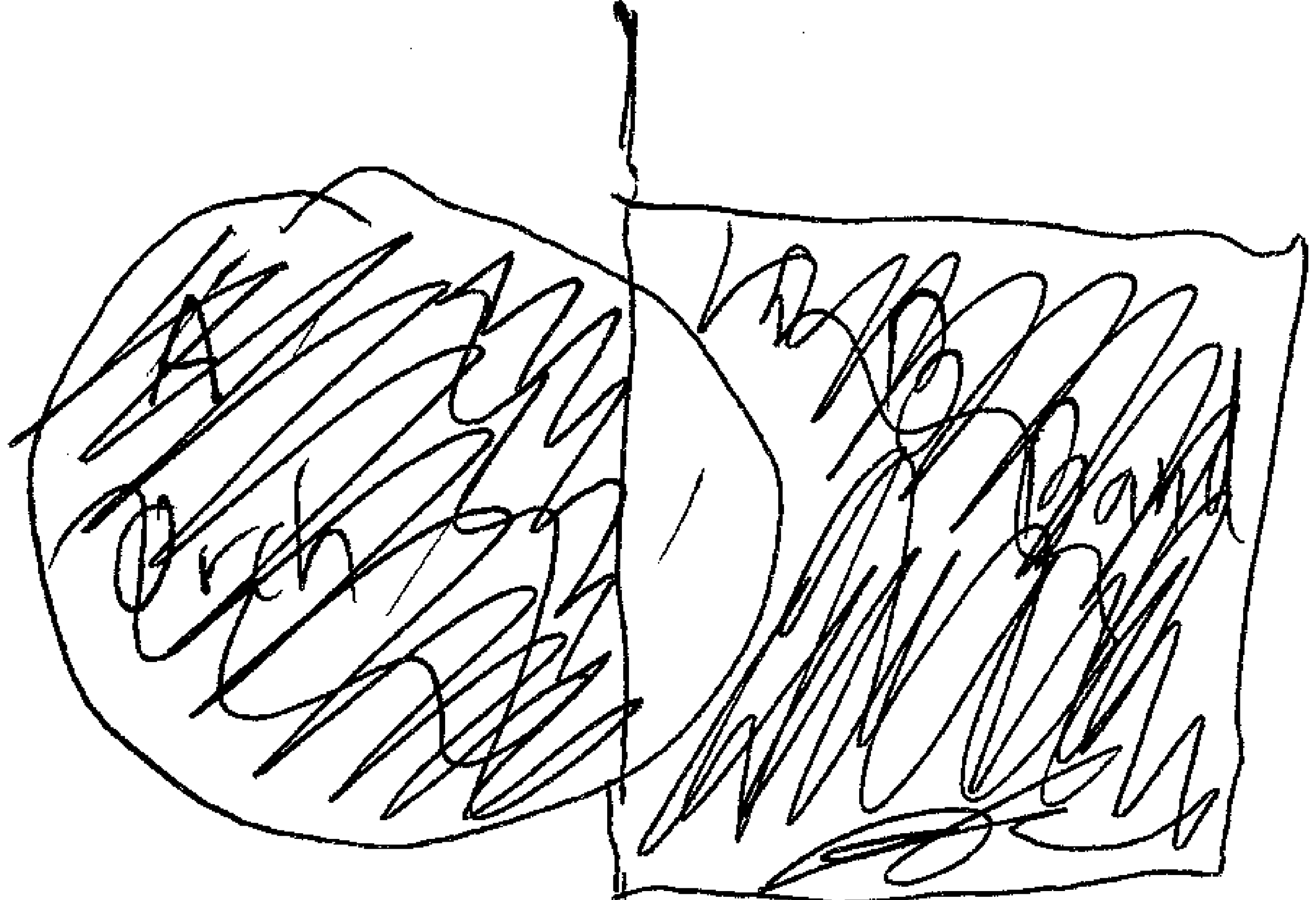
$\sim B$ $\tilde{B}$
complement of set.
$\overline{B}$ B'

In Orch, Not in Band.

$= A \cap (\sim B)$

$A - B$

$A \setminus B$ "Backslash Set Minus"



People in One but not the other ②

$= (A \setminus B) \cup (B \setminus A)$

Alt Notations: $A \oplus B$, $A \triangle B$.

$A \triangle B \triangle B \stackrel{?}{=} A$

$A \oplus B \oplus B \stackrel{?}{=} A$

6 people

4 in orch

3 in Band

(1 in neither)

Orch = {1, 2, 4, 6}

Band {1, 4, 5}

neither = 3

	1	2	3	4	5	6	
A Orch =	1	1	0	1	0	1	
Band	1	0	0	1	1	0	
Band ∪ Orch	1	1	0	1	1	1	Max
Band ∩ Orch	1	0	0	1	0	0	Bitwise And = Bitwise Min
Orch Δ Band	0	1	0	0	1	1	Bitwise XOR
= A Δ B Δ B	1	0	0	1	1	0	B
= A	1	1	0	1	0	1	= Orch Again

Vector Addn Mod 2

~~Continue from Thu 1/29~~ overhead notes --

$\Sigma = \{0, 1\}$: object of type set<char> = Alph.

Sigma = {'0', '1'}   '0' : char

"0" : string

Bin = $\mathcal{B}$ = {"0", "1"} : object of type set<string>

: Language

B_2 = {"00", "01", "10", "11"}   Empty string: "" or ε (epsilon) or λ (lambda)

$= \{00, 01, 10, 11\} = \Sigma^2 = \{0,1\}^2$

$B_3 = \{000, 001, 010, 011, 100, 101, 110, 111\} = \Sigma^3 = \{0,1\}^3$

The set of all finite binary strings is written Σ^* $*$ ≡ zero or more.

superscript $+$ ≡ one or more.

$B_2' = \{\varepsilon, 0, 1, 00, 01, 10, 11\} = \text{"}\Sigma^{\leq 2}\text{"}$

Pairs are any elements (a,b) of a Cartesian Product $A \times B$

$(a,b) \in A \times B$ means $a \in A$ & $b \in B$.

When $A = B$, we can abbreviate $A \times A$ as A^2. Thus

$\Sigma^2 = \Sigma \times \Sigma = \{(0,0), (0,1), (1,0), (1,1)\}$ "collapsed" as $\{00, 01, 10, 11\}$

Triples (a,b,c) belong to $A \times B \times C \equiv (A \times B) \times C \equiv (A \times (B \times C))$

$A \times A \times A \equiv A^3$ and $\Sigma^{\leq 3} = \Sigma^3 \cup \Sigma^2 \cup \Sigma \cup \underbrace{\{\varepsilon\}}_{\Sigma^0}$

Relations: A binary relation R between sets A and B is any subset of $A \times B$.

Empty relation: $\emptyset$
Full relation: $A \times B$ itself.

If $B = A$, we call it a binary relation on A.

Example: $R_{id} = \{(a,a) : a \in A\}$. ¿Does this equal $A \times A$? Generally $\boxed{\text{NO}}$

$A = \Sigma = \{0,1\}$. $R_{id} = \{\underset{00}{(0,0)}, \underset{11}{(1,1)}\}$. $A \times A$ includes $\underset{01}{(0,1)}, \underset{10}{(1,0)}$

A relation R is a function if for all $a \in A$ there is (exactly) one $b \in B$ such that $(a,b) \in R$. R_{id} = identity function.

"at most" defines a partial function whose actual domain can be a subset of A.

Usually: $f : A \to B$ means $\boxed{\begin{array}{l} Dom(f) = A \\ Ran(f) \subseteq B \end{array}}$.

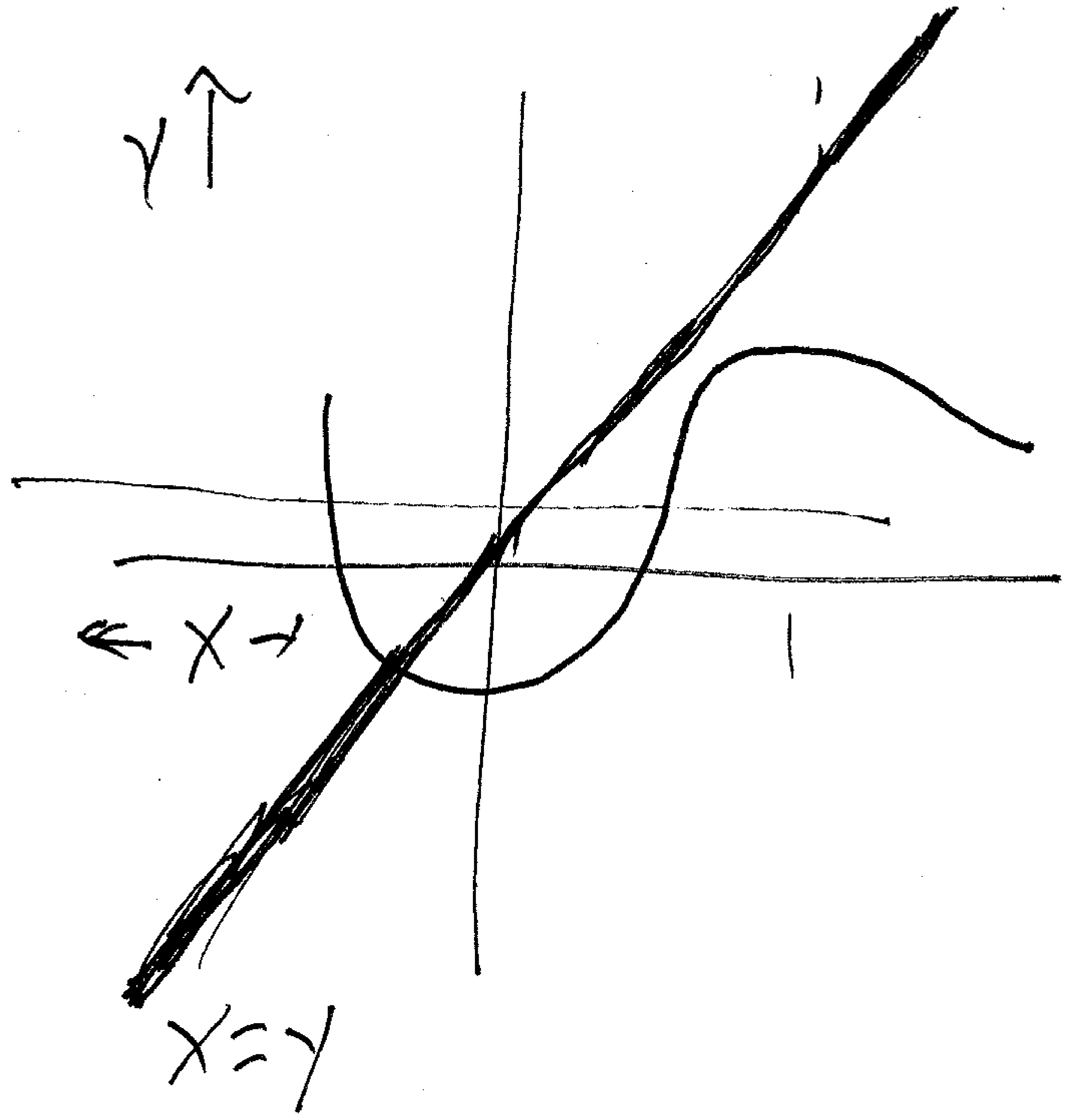
But sometimes we lie and allow $Dom(f) \subsetneq A$.
Then f is a partial function.

Defn: f is a partial function if for all $a \in A$ there is at most one $b \in B$ such that (a,b) belongs to the relation of f.

Is $\emptyset$ a partial fn? [from any A to any B]

But: $\emptyset : \underset{A}{\emptyset} \to \underset{B}{\emptyset}$ as a proper total function!

R defines a total function if $(\forall a \in A)(\exists b \in B) : (a,b) \in R$.
R is onto its "declared range" if $(\forall b \in B)(\exists a \in A) : (a,b) \in R$.
R is 1-1 if $\cancel{(\forall a \in A)(\exists \text{ at most one } b \in B)} : (a,b) \in R$
$(\forall b \in B)(\exists \text{ at most one } a \in A) : (a,b) \in R$.



Defn: $R \subseteq A \times A$ is an equivalence relation if it is:

Reflexive: $\forall a \in A,\ (a,a) \in R$.

Symmetric: $\forall a,b \in A\ (a,b) \in R \leftrightarrow (b,a) \in R$.

Transitive: $\forall a,b,c \in A$: $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

R(story v1, story v2) ≡ Versions 1 and 2 are the same up to 95% of the words in either.

R'(story 1, story 2) ≡ Stories 1 & 2 differ only in their headlines. at most.

Yes, this is an equivalence relation.

Prove: All horses have the same color.

≡ All finite sets of horses have the same color.

For all $n \geq 0$, all sets of n horses have the same color.

Basis: $(n=1)$ Any set of one horse has just one color.

Ind: Given $n \geq 2$, assume (Ind. Hyp.) all sets of $n-1$ horses have the same color.

Let any set S of horses be given. List S out as $\{H_1, H_2, \ldots, H_n\}$

Take $S_1 = \{$first $n-1$ horses$\} = \{H_1, H_2, \ldots, H_{n-1}\}$

$S_2 = \{$last $n-1$ horses$\} = \{H_2, H_3, \ldots, H_n\}$. QED! (???)