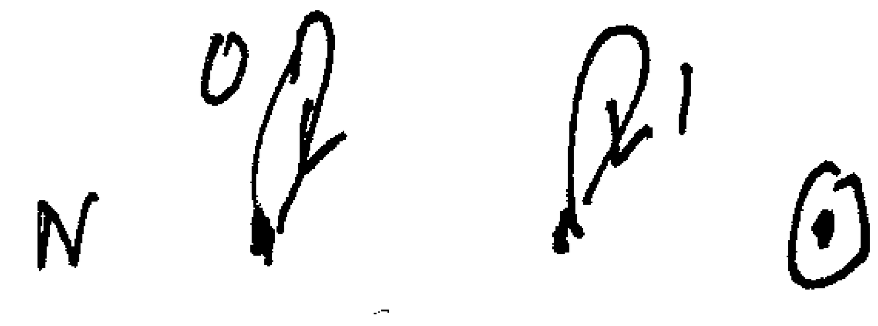
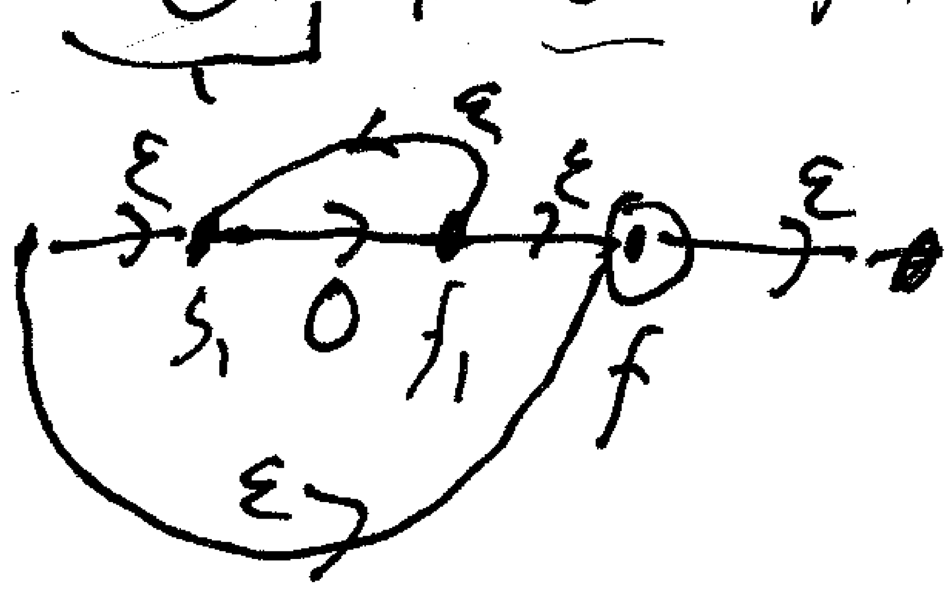
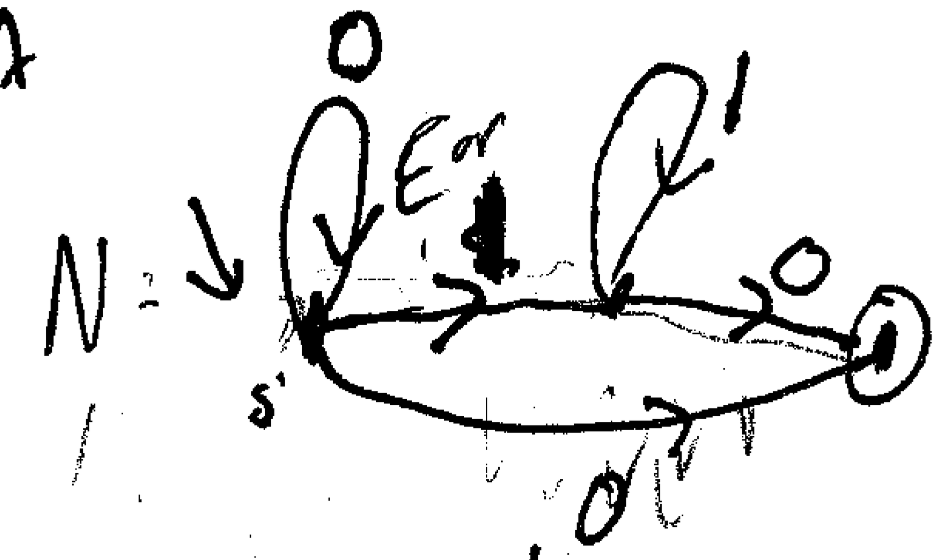
[Lecture Thursday 2/19]

Examples of Regexp → NFA and NFA → DFA



$r = 0^* \cdot 1^* \cdot 0$ Better NFA





Is N *sound*, meaning Is $L(N) \subseteq L(r)$?

Is N *comprehensive*, meaning $L(r) \subseteq L(N)$, [so $L(r) = L(N)$]?

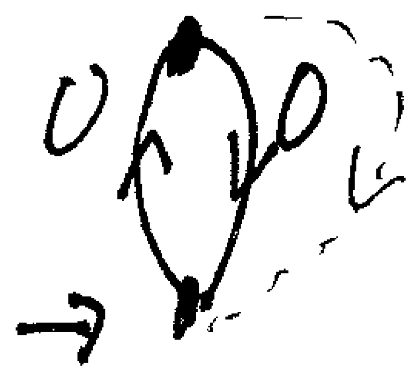
How can we further economize this NFA? (If we care...)

We have a 3-state NFA (either of the three ways)

But it is not a DFA.

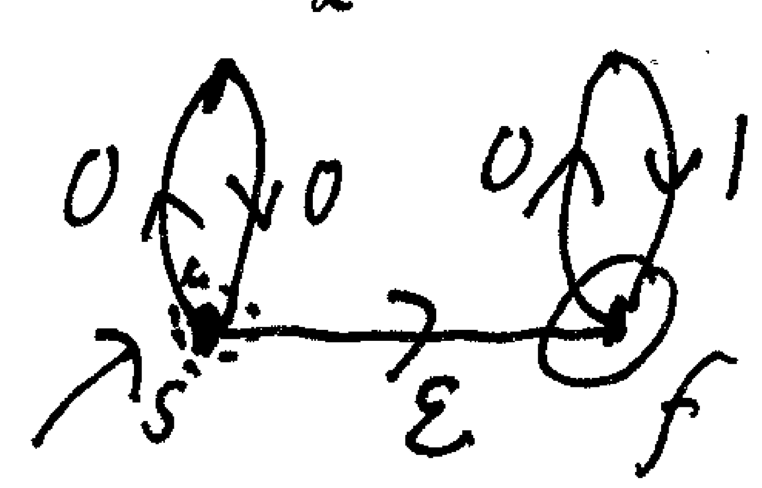
- no dead state (when it is reachable)
- nondeterminism at S.

$r = (00)^* (01)^*$



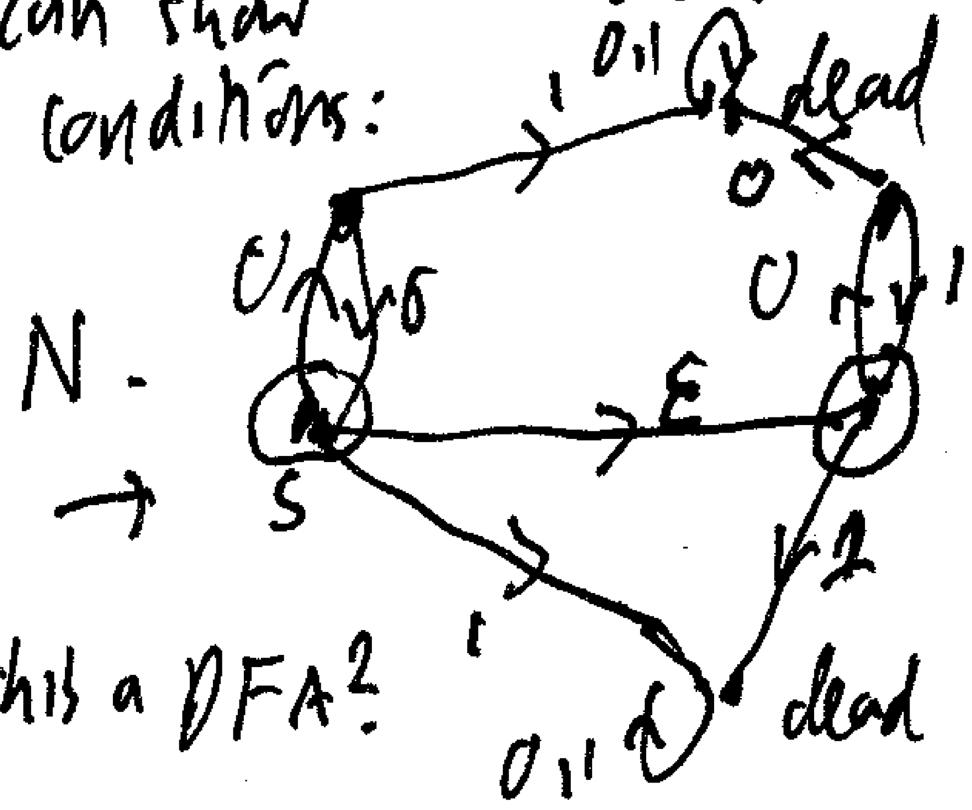
Unsound: allows 0100

Hence • needs a real ε



Can put $s \in F$ too

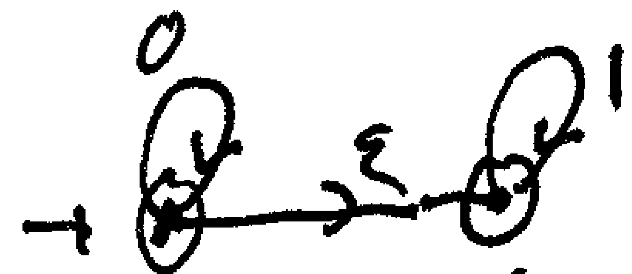
We can show dead conditions:



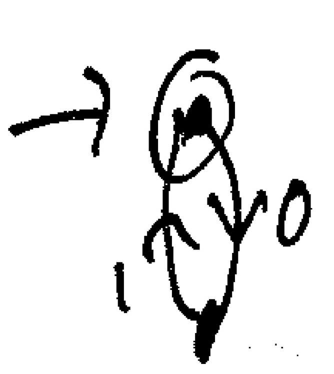
Is this a DFA?

No: real nondeterminism on 0 at start.

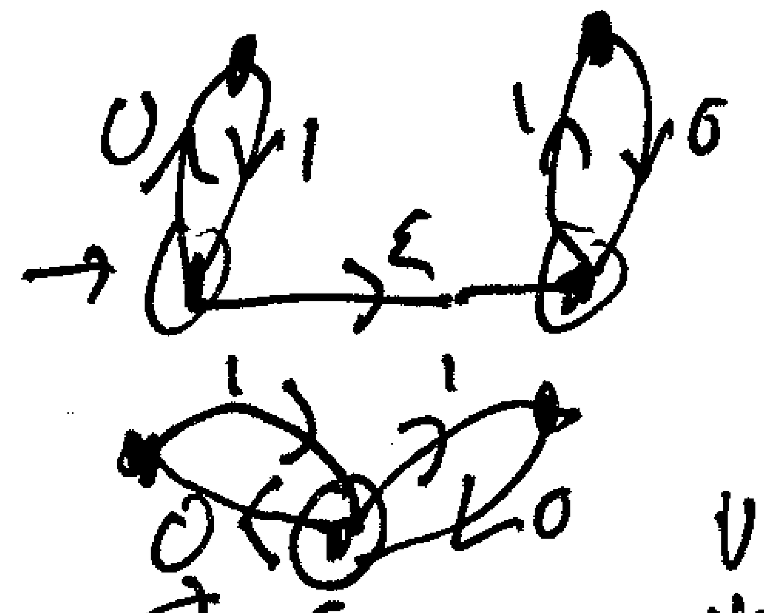
$0^* 1^*$



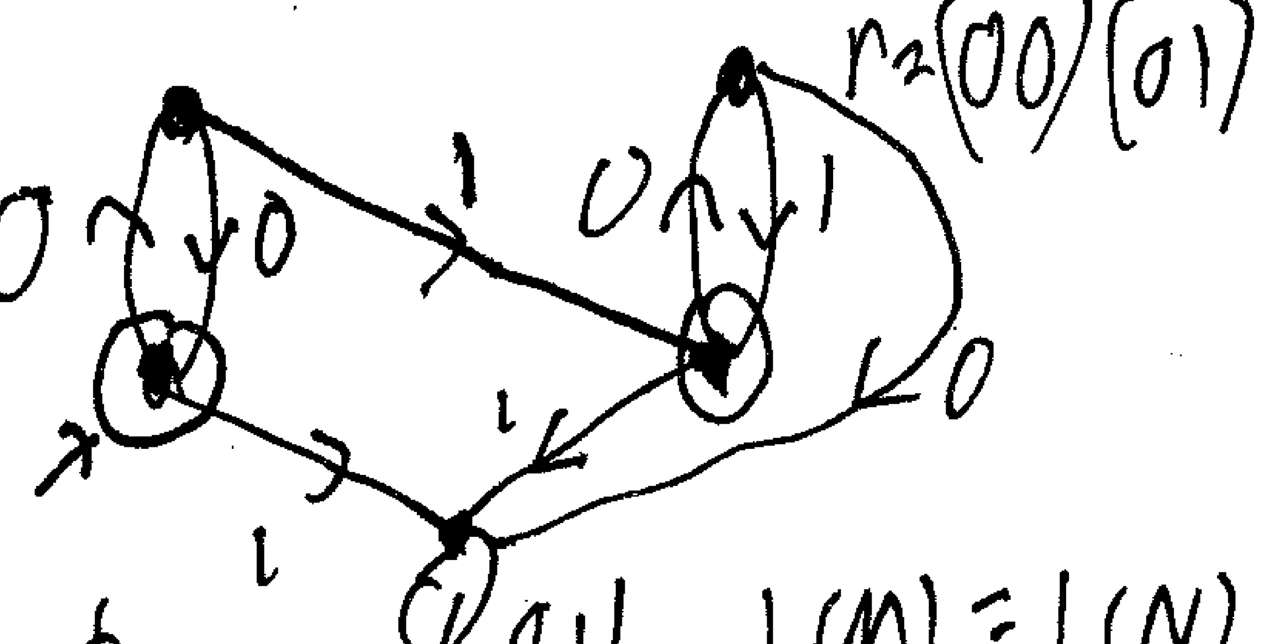
$(01)^*$



$L((01)^*(10)^*)$ $\cap$ $L((01+10)^*)$

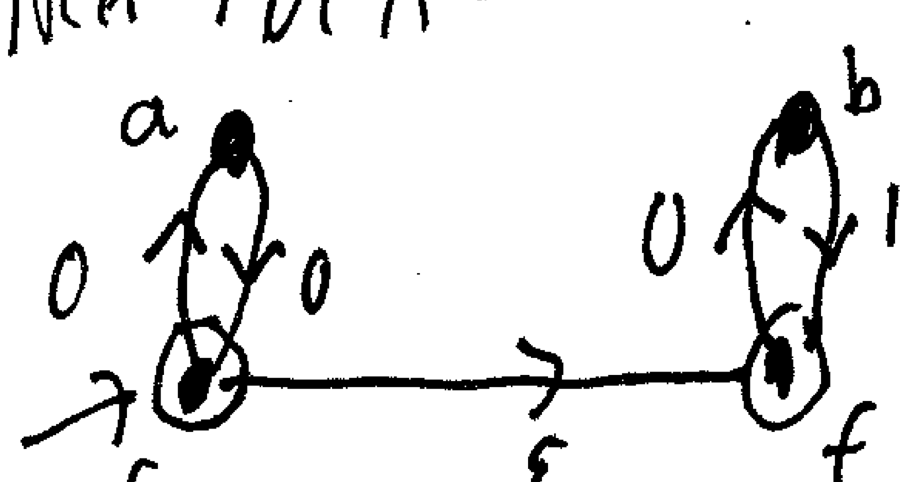


Unlike the NFA with ε now we NEED $s \in F$!



= a DFA M s.t. $L(M) = L(N)$

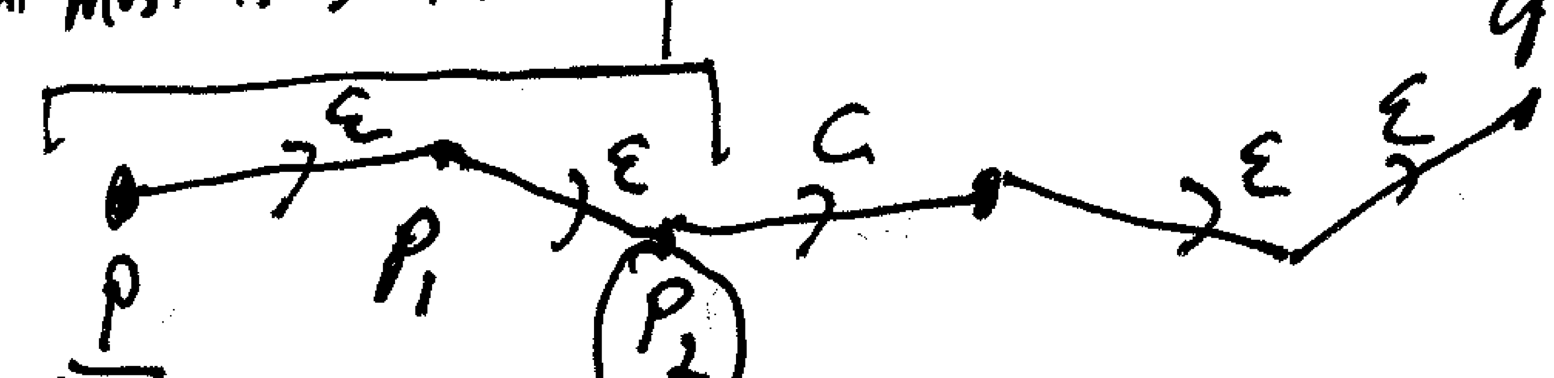
NFA → DFA:



"Whenever s, then also f."

∴ The combinations $\{s\}$, $\{s,a\}$, $\{s,b\}$ and $\{s,a,b\}$ never happen. ∴ DFA has at most 12 states.

$\Delta(P,c) = \{q$: for some $p \in P$, N can process c from p to q$\}$.



We will maintain the invariant that P is ε-closed, meaning it includes all states reachable by ε-arcs from other states in P.

Doing so allows us to concentrate on processing c immediately, then any ε's.

$\delta(s,0) = \{a\}$

$\delta(s,1) = \emptyset$

$\delta(a,0) = \{s,f\}$

$\delta(a,1) = \emptyset$

$\delta(b,0) = \emptyset$

$\delta(b,1) = \{f\}$

$\delta(f,0) = \{b\}$

$\delta(f,1) = \emptyset$.

$\Delta(P,c) = \bigcup_{p \in P} \delta(p,c)$

Since our start state S is $\{s,f\}$,

$\Delta(S,0) = \delta(s,0) \cup \delta(f,0)$

$= \{a\} \cup \{b\} = \{a,b\}$.

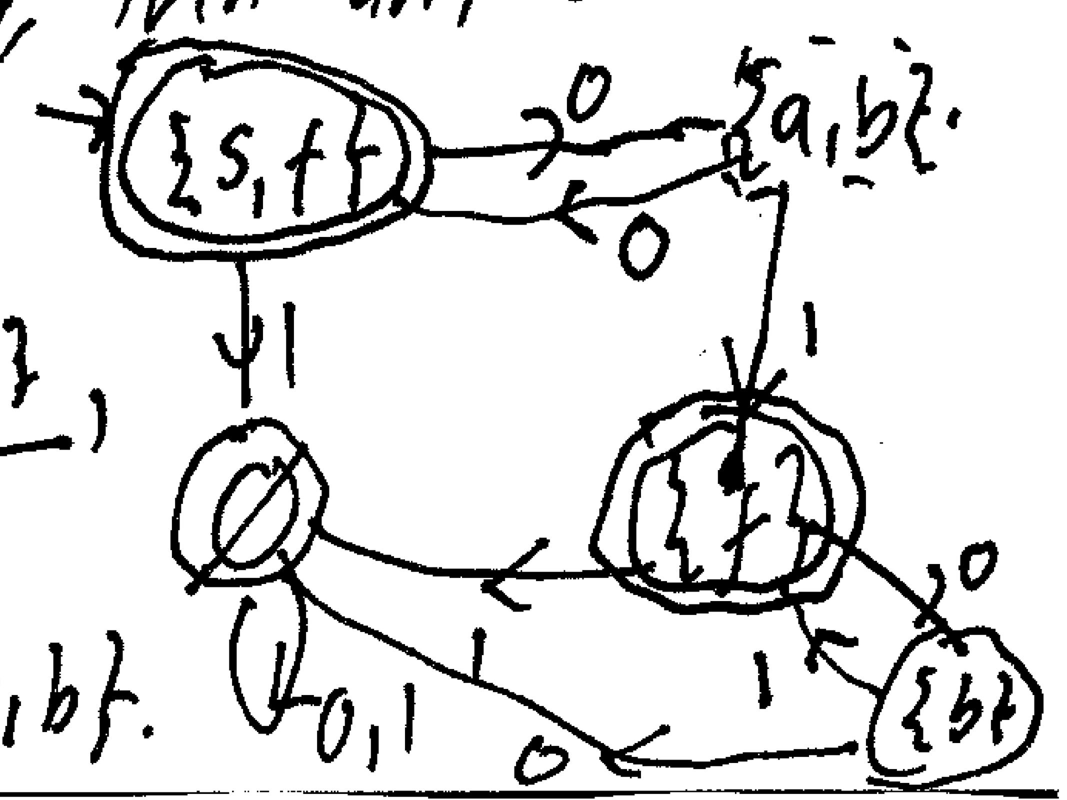
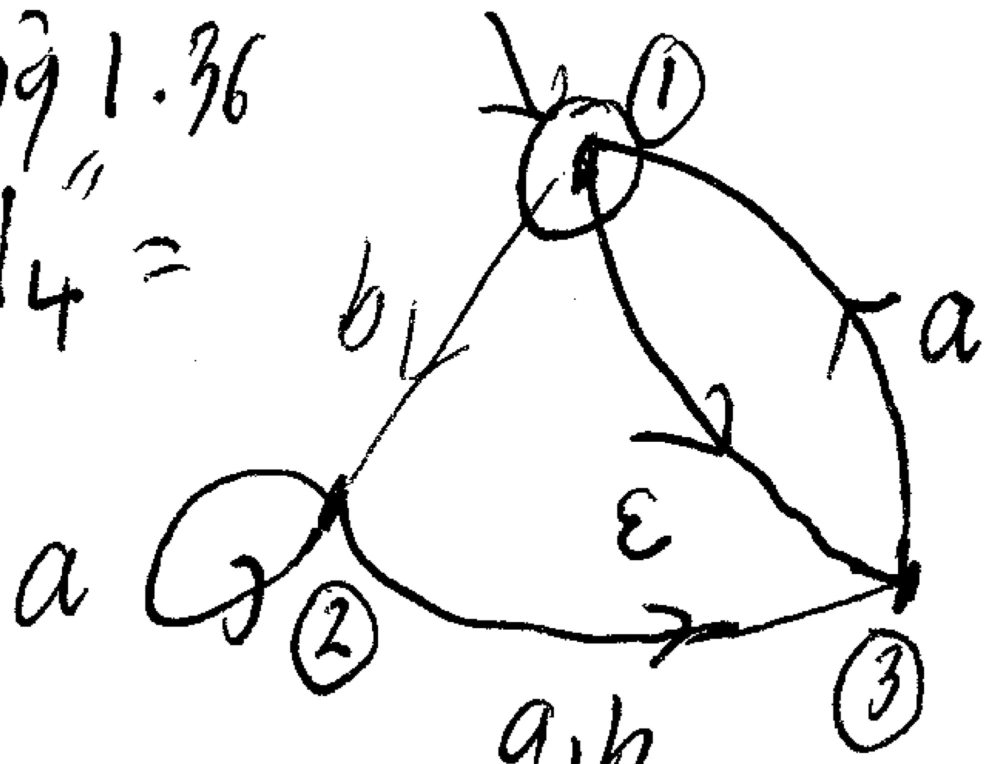


Fig 1.36

"N_4" =



"whenever 1, also 3."

$S = \{1,3\}$, not $\{1\}$!

F = any set containing ①.

$\underline{\delta}(1,a) = \emptyset$

$\underline{\delta}(1,b) = \{2\}$

$\underline{\delta}(2,a) = \{2,3\}$

$\underline{\delta}(2,b) = \{3\}$

$\underline{\delta}(3,a) = \{1,3\}$

$\underline{\delta}(3,b) = \emptyset$

$\Delta(S,a) = \Delta(\{1,3\},a) = \underline{\delta}(1,a) \cup \underline{\delta}(3,a)$
$= \emptyset \cup \{1,3\} = \{1,3\}$.

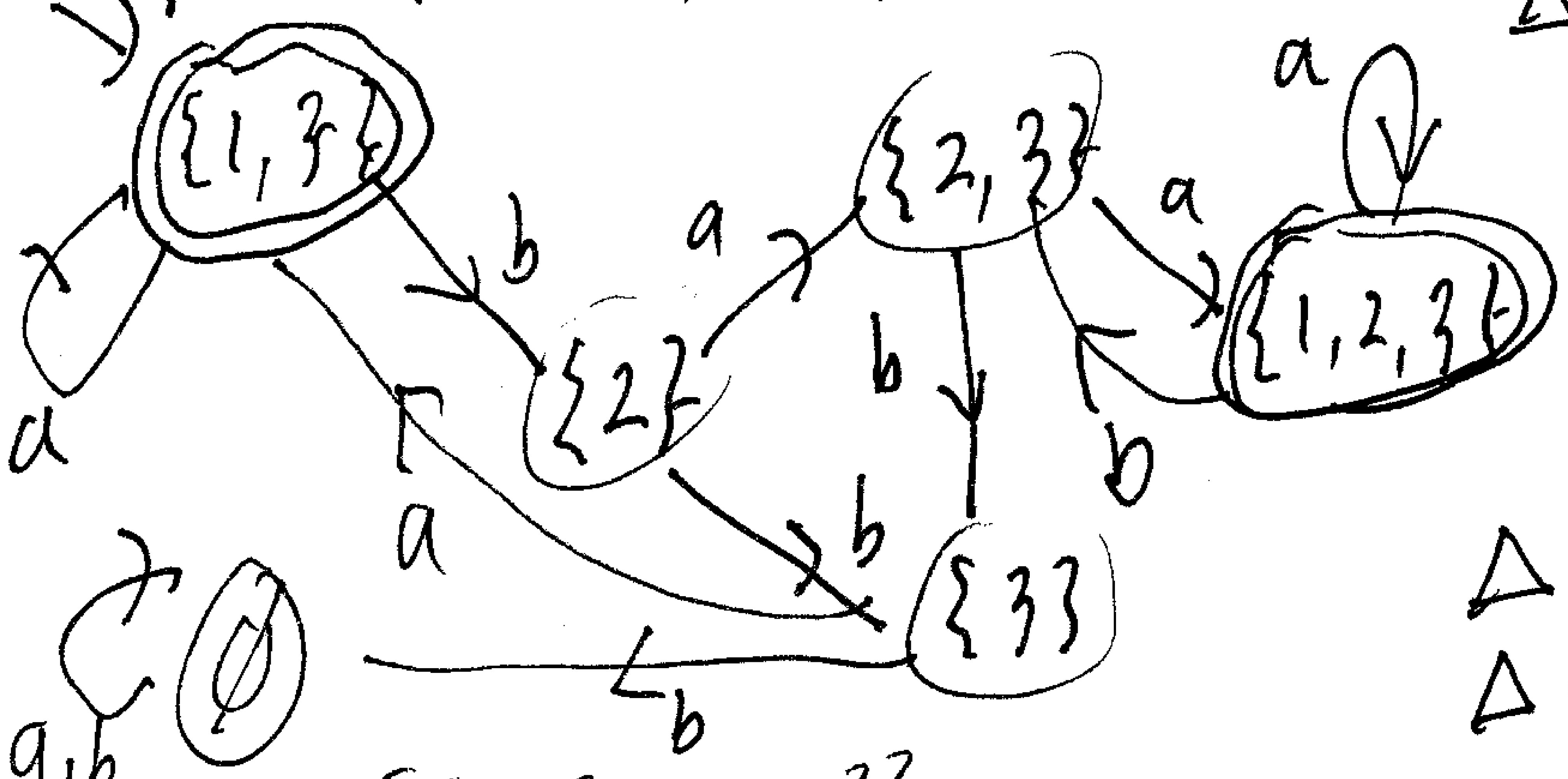
$\Delta(S,b) = \underline{\delta}(1,b) \cup \underline{\delta}(3,b)$
$= \{2\} \cup \emptyset = \{2\}$

$\Delta(\{2\},a) = \underline{\delta}(2,a) = \{2,3\}$

$\Delta(\{2\},b) = \underline{\delta}(2,b) = \{3\}$

$\Delta(\{2,3\},a) = \underline{\delta}(2,a) \cup \underline{\delta}(3,a)$
$= \{2,3\} \cup \{1,3\} = \{1,2,3\}$

$\Delta(\{2,3\},b) = \underline{\delta}(2,b) \cup \underline{\delta}(3,b)$
$= \{3\} \cup \emptyset = \{3\}$



added $\rightarrow F = \{\{1,3\}, \{1,2,3\}\}$

$\Delta(\{3\},a) = \{1,3\}$

$\Delta(\{3\},b) = \emptyset$

$\Delta(\{1,2,3\},b) = \underline{\delta}(1,b) \cup \underline{\delta}(2,b) \cup \underline{\delta}(3,b)$
$= \{2\} \cup \{3\} \cup \{\emptyset\} = \{2,3\}$

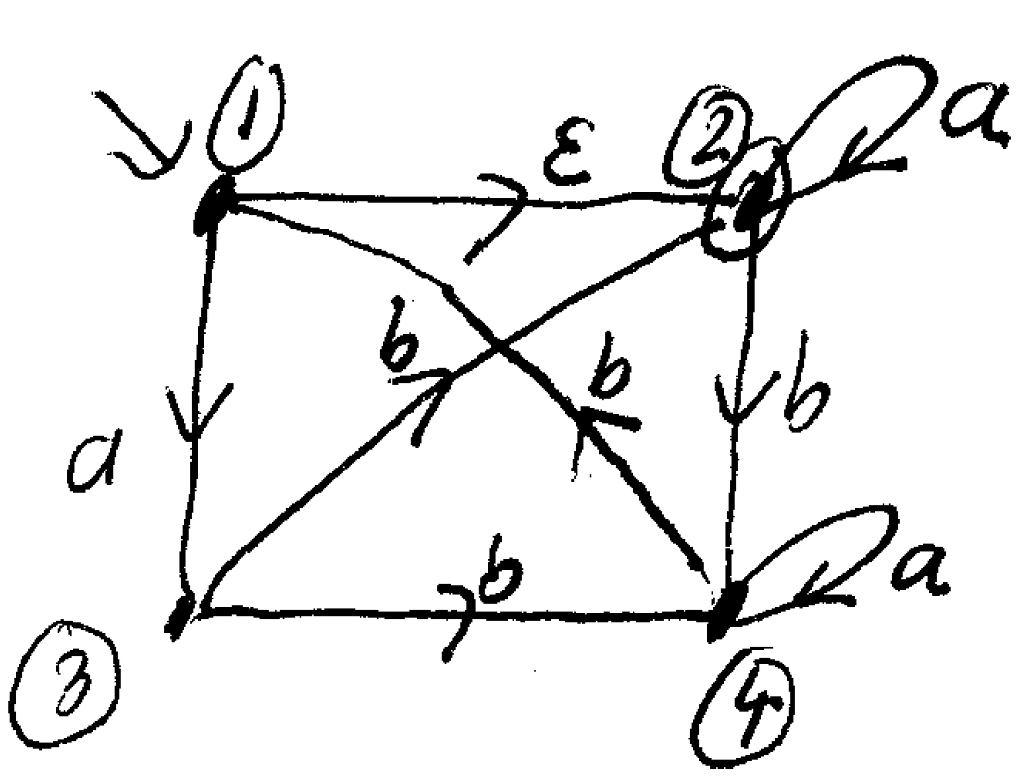
$\boxed{\text{The lecture ended here.}}$

<u>Extra Material</u>:

The text's proof of Theorem 1.39 gives the formula $\Delta(R,a) = \bigcup_{r \in R} \delta(r,a)$ (text: δ'_N) <u>only</u> for the case <u>without</u> ε-arcs. Then my "$\underline{\delta}$" coincides with the text's function view of "δ". But in the ε-arc case the text does a "patch" on the next page using $E(R)$ to mean the "ε-closure" of R. The modified rule for $\delta'(R,a)$ is (IMHO) harder to visualize. What my "$\underline{\delta}$" does is accomplish the ε-closure <u>up-front</u>. Then the union rule $\Delta(R,a) = \bigcup_{r \in R} \underline{\delta}(r,a)$ is more straightforward to use — less error-prone. Try it anyway and see ...

A diagram of the NFA N on problem set 3, problem 2:

$s = 1$, $F = \{2\}$



$\delta = \{(1,\varepsilon,2), (1,a,3),$
$(2,a,2), (2,b,4),$
$(3,b,2), (3,b,4),$
$(4,a,4), (4,b,1)\}$.

The HW also asks you to write a regular expression for L_{11}. Here L_{11} is a subset of $L_{12} = L(N)$ because of the ε-arc, but L_{12} includes other strings. In fact, $L_{12} = L_{11} \cdot (\varepsilon + ab) \cdot a^*$ — can you "read" that from the diagram?