Knowledge Representation and Reasoning

Logics for Artificial Intelligence

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Contents

1. Introduction 3

2. Propositional Logic 15

3. Elementary Predicate Logic 116

4. Full First-Order Predicate Logic 159

5. Summary of Part I 259

6. Prolog 267

7. A Potpourri of Subdomains 286

7 A Potpourri of Subdomains

7.1 Taxonomies 286
7.2 Time 291
7.3 Things *vs.* Substances 299

Taxonomies: Categories as Intensional Sets

In mathematics, a set is defined by its members.

This is an **extensional set**.

Plato: *Man is a featherless biped.*

An **intensional** set is defined by properties.

Aristotle: *Man is a rational animal.*

A category (type, class) is an intensional set.

Taxonomies: Need for Two Relations

With sets, there's a difference between

set membership, $\in$
$5 \in \{1,3,5,7,9\}$

and subset, $\subset, \subseteq$
$\{1,3,5,7,9\} \subset \{1,2,3,4,5,6,7,8,9\}$

One difference is that subset is transitive, membership isn't.

Taxonomies: Categories as Unary Predicates

One way to represent taxonomies:

$Canary(Tweety)$

$\forall x[(Canary(x) \Rightarrow Bird(x)]$
$\forall x[(Bird(x) \Rightarrow Vertebrate(x)]$
$\forall x[(Vertebrate(x) \Rightarrow Chordate(x)]$
$\forall x[(Chordate(x) \Rightarrow Arthropod(x)]$
$\forall x[(Arthropod(x) \Rightarrow Animal(x)]$

Taxonomies: Reifying

To reify: to make a thing of.
Allows discussion of "predicates" in FOL.

Membership: Member or Isa
$\mathit{Isa}(\mathit{Tweety}, \mathit{Canary})$

Subcategory: $\mathit{Subclass}$ or Ako (sometimes, even, Isa)
$\mathit{Ako}(\mathit{Canary}, \mathit{Bird})$
$\mathit{Ako}(\mathit{Bird}, \mathit{Vertebrate})$
$\mathit{Ako}(\mathit{Vertebrate}, \mathit{Chordate})$
$\mathit{Ako}(\mathit{Chordate}, \mathit{Arthropod})$
$\mathit{Ako}(\mathit{Arthropod}, \mathit{Animal})$

Rules:
$\forall x \forall c_1 \forall c_2 [\mathit{Isa}(x, c_1) \land \mathit{Ako}(c_1, c_2) \Rightarrow \mathit{Isa}(x, c_2)]$
$\forall c_1 \forall c_2 \forall c_3 [\mathit{Ako}(c_1, c_2) \land \mathit{Ako}(c_2, c_3) \Rightarrow \mathit{Ako}(c_1, c_3)]$

Transitive Closure

It's sometimes useful (especially in Prolog) to have a second relation, R_2 be the transitive closure of a relation, R_1.

E.g. $ancestor$ is the transitive closure of $parent$:

$$\forall x, y[parent(x, y) \Rightarrow ancestor(x, y)]$$

$$\forall x, y, z[parent(x, y) \wedge ancestor(y, z) \Rightarrow ancestor(x, z)]$$

Discussing Categories

$$Isa(Canary, Species)$$
$$Isa(Bird, Class)$$
$$Isa(Arthropod, Phylum)$$
$$Isa(Animal, Kingdom)$$

$$Extinct(Dinosaur)$$

Note: That's Isa, not Ako.

If categories are predicates, requires second-order logic.

Other relationships: exhaustive subcategories, disjoint categories, partitions, "tangled hierarchies" (separate groups of disjoint subcategories).

Time

How would you represent time?

Discuss

Subjective vs. Objective: Subjective

Make now an individual in the domain.

Include other times relative to now.

OK if time doesn't move.

Subjective vs. Objective: Objective

Make `now` a meta-logical variable with some time-denoting term as value.

Relate times to each other, *e.g.* $Before(t1, t2)$.

Now can move by giving `now` a new value.

Points vs. Intervals: Points

Use numbers: integers, rationals, reals?

Computer reals aren't really dense.

How to assign numbers to times?

Granularity: How big, numerically, is a day, or any other interval of time?

If an interval is defined as a pair of points, which interval is the midpoint in, if one interval immediately follows another?

Points vs. Intervals: Intervals

Use intervals only: no points at all.

More cognitively accurate.

Granularity is not fixed.

A “point” is just an interval with nothing inside it.

James Allen's Interval Relations

```
                           y
before(x,y)  |---|  |---|
               x
```

```
                          y
meets(x,y)   |---|---|
               x
```

```
                      y
overlaps(x,y)       |----|
                  |----|
                     x
```

```
                 y
               |---|
equals(x,y)    |---|
                 x
```

```
                     y
starts(x,y)        |-----|
                   |---|
                     x
```

```
                       y
finishes(x,y)    |-----|
                   |---|
                     x
```

```
                   y
during(x,y)      |-----|
                  |---|
                   x
```

A Smaller Set of Temporal Relations

If fewer distinctions are needed, one may use

$\mathit{before}(x, y)$ for Allen's $\mathit{before}(x, y) \vee \mathit{meets}(x, y)$

$\mathit{during}(x, y)$ for Allen's $\mathit{starts}(x, y) \vee \mathit{during}(x, y) \vee \mathit{finishes}(x, y)$

$\mathit{overlaps}(x, y)$ and $\mathit{equals}(x, y)$

and appropriate converses.

Things vs. Substances

Count Nouns vs. Mass Nouns

A count noun denotes a thing.
Count nouns can be singular or plural.
Things can be counted.
One dog. Two dogs.

A mass noun denotes a substance.
Mass nouns can only be singular.
One can have a quantity of a substance.
A glass of water. A pint of ice cream.

A Quantity of a Substance is a Thing

water a substance

a lake = *a body of water* a thing

lakes a plurality of things

40 acres of lakes a quantity of a substance

Nouns with mass and count senses

A noun might have both senses.

a piece of pie vs. *A piece of a pie*

two pieces of steak vs. *two steaks*

Any count noun can be "massified".
Any thing can be put through "the universal grinder".
I can't get up; I've got cat on my lap.